Distinguishing Constraints on Financial Inclusion and Their Impact on GDP, TFP, and the Distribution of Income

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July 5, 2018

Abstract

We develop a tractable general equilibrium model with heterogeneous agents to study how different sources of financial frictions interact in equilibrium. We examine the differential impact of alleviating these frictions on GDP and TFP through intensive and extensive margins. Alleviating each constraint has different quantitative effects and can result in welfare losses for some agents in general equilibrium. When multiple financial constraints are alleviated together, the impact through the intensive margin is amplified but the effect through the extensive margin is dampened. Our paper demonstrates the importance of distinguishing the sources of financial frictions to help design effective country-specific financial inclusion policies.

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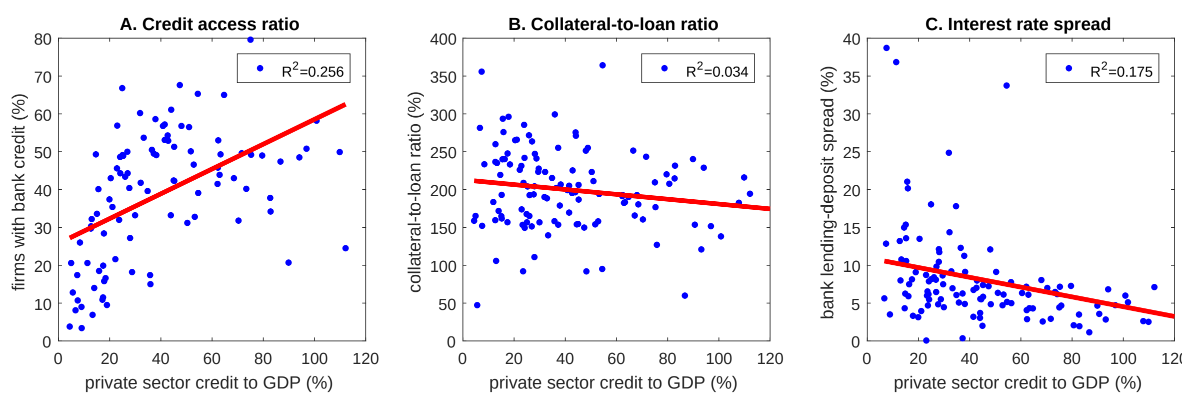
1 Introduction

Financial deepening has accelerated in emerging market economies and low-income countries over the past two decades. The record on financial inclusion, however, has not kept apace. Large amounts of credit do not always correspond to broad use of financial services, as credit is often concentrated among the largest firms. As a result, firms in developing countries continue to face barriers in accessing financial services.

Financial system development is multi-faceted in nature. The stage of financial development can be captured by different indicators, credit access ratio, collateral-to-loan ratio, and interest rate spread, corresponding to the breadth (ability of firms to access bank credit), depth (the amount of collateral required for borrowing), and efficiency (ability to provide financial intermediation at low cost) of a financial system. In general, countries with deeper financial institutions (as commonly proxied by the high credit-to-GDP ratio) tend to have high breadth, depth, and efficiency. However, there exist large difference across countries in the source of financial frictions (see Figure 1). Given that financial deepening is multi-dimensional, involving both participation barriers and financial frictions that constrain credit availability, policies to foster financial inclusion are likely to vary across countries.



Source: the World Bank Enterprise Surveys and Development Indicators. Among the 150 countries in our sample, the correlation is -0.11 between credit access ratio and interest rate spread, -0.32 between credit access ratio and collateral-to-loan ratio, and -0.02 between interest rate spread and collateral-to-loan ratio.

Figure 1: Correlations between the private credit to GDP ratio and various financial system indicators.

These considerations warrant a tractable framework that allows for a systematic examination of the linkages between financial deepening, GDP, TFP, and inequality. In this paper, we develop a micro-founded general equilibrium model to highlight, distinguish, and evaluate the differential impacts of different financial constraints on GDP, TFP, and inequality and examine how these constraints interact both theoretically and numerically.

In the model, agents are heterogeneous—distinguished from each other by wealth and entrepreneurial productivity. In each period, agents choose whether to become entrepreneurs or workers. Workers supply labor to entrepreneurs and are paid the equilibrium wage. Entrepreneurs have access to a technology that uses capital and labor for production. In equilibrium, only productive agents with a certain level of wealth choose to become entrepreneurs. Unproductive agents, or those who are productive but wealth constrained, are unable to start a profitable business, choosing instead to become wage earners.

The model features an economy with two "financial regimes", one with credit and one with savings only. Agents in the savings regime can save but cannot borrow. Participation in the savings regime is free, but agents must pay a credit entry cost to receive loans (Greenwood and Jovanovic, 1990; Townsend and Ueda, 2006). The credit entry cost is one of the determinants of financial inclusion, capturing the fixed transactions costs and high annual fees, documentation requirements, and other access barriers facing entrepreneurs in developing countries. Once in the credit regime, agents can obtain credit, but its size is constrained by two additional types of financial frictions. The first financial friction is modelled as a collateral constraint, which arises from imperfect enforceability of contracts. Entrepreneurs have to post collateral in order to borrow. The value of collateral is thus another determinant of financial inclusion, affecting the amount of credit available. The second financial friction arises from the intermediation cost from banks, which could be due to the information cost as in Townsend (1979) and Greenwood, Sanchez and Wang (2013).

The three types of financial frictions distort the allocation of capital and entrepreneurial talent. Financial inclusion refers to the policies that alleviate these financial constraints, reflected in high credit entry costs, collateral requirements, and intermediation costs. These frictions are empirically relevant. For example, the existing literature has documented that the distance to a bank branch matters for credit access, suggesting that policies to promote branch openings in rural unbanked locations could help reduce the credit entry cost in our model.[1] Moreover, during the 2007-2009 financial crisis, many countries relaxed collateral constraints by widening the range of securities that could be accepted

[1] See Assuncao, Mityakov and Townsend (2012); Alem and Townsend (2013); Gilje, Loutskina and Strahan (2016); Nguyen (2017); Aguirregabiria, Clark and Wang (2017); Celerier and Matray (2017) for evidence. Many developing countries have conducted such kind of policies to increase credit access. For example, after a bank nationalization in 1969, the Indian government launched an ambitious social banking program which sought to improve the access of the rural poor to formal credit and savings opportunities (Burgess and Pande, 2005).

as collateral with the aim of boosting lending to firms and households. Finally, financial deepening can lead to increased competition among financial institutions, accelerating investment in computerization, thereby improving intermediation efficiency and lowering intermediation costs.[2]

In a partial equilibrium analysis with fixed interest rates and wages, we show that relaxing the ex-ante friction captured by the credit entry cost and the two ex-post frictions within the credit regime can increase GDP through both the extensive margin (i.e. moving entrepreneurs from the savings regime to the credit regime) and the intensive margin (i.e. increasing output of entrepreneurs in the credit regime). Importantly, we find that there are non-trivial interactions among the three financial constraints. The credit entry cost, the collateral constraint, and the intermediation cost have amplifying effects on the intensive margin, but have dampening effects on the extensive margin. Intuitively, this is because a lower entry cost increases the credit access ratio, such that relaxing collateral constraints and lowering intermediation costs have less of an impact. In other words, when the credit entry cost is low, the credit access ratio is already high, so that there is little room for further increasing access through the other two channels. On the intensive margin, relaxing one constraint amplifies the impact of relaxing other constraints. This is because, when the credit entry cost is low, entrepreneurs are now left with more wealth after entering the credit regime. Since the amount of credit and the total intermediation cost are proportional to wealth, relaxing collateral constraints and lowering intermediation costs increase business profits more.

Deriving the general equilibrium effect of these policies does not allow for analytical solutions, since the wealth-productivity distribution and equilibrium interest rates and wages are endogenous. To better understand the differential impacts of relaxing the various financial constraints, and in particular, how they interact in general equilibrium, we calibrate the model using data from the World Bank Enterprise Surveys and World Development Indicators. We jointly choose the model's key parameters to match the data moments for the Phillipines, such as the percent of firms with credit, the collateral-to-loan ratio, interest rate spread, income inequality, gross savings rate, and the firm employment distribution.

We first examine the aggregate and distributional implications of financial inclusion. We compare GDP, TFP, income Gini coefficient, and welfare across different steady states. Although relaxing the three constraints all lead to higher GDP and TFP, the effects are

[2] For example, from 1985 to 1994, the Thai banking sector had become a more capital-intensive industry, substituting physical capital for labor. The average cost of raising funds decreased from 14.40% in 1985 to 5.61% in 1994 for large-sized banks (Okuda and Mieno, 1999).

channeled through different margins. Specifically, when the credit entry cost is reduced, GDP and TFP initially increase mostly through the intensive margin. This is because most entrepreneurs do not have sufficient wealth to pay the upfront cost and are still excluded from the credit market. When the credit entry cost is further lowered, the extensive margin starts to play a role. By contrast, we find that when the collateral constraint is relaxed, both intensive and extensive margins contribute significantly to GDP and TFP growth. This is because relaxing the collateral constraint allows entrepreneurs to employ more capital in the credit regime, thus motivating them to pay the credit entry cost in the first place. When the intermediation cost is lowered, GDP and TFP growth is attributed to the extensive margin (but not the intensive margin) as the scale of business in the credit regime is constrained by the collateral constraint.

In terms of income inequality, reducing the credit entry cost and relaxing the collateral constraint initially increases the Gini coefficient. But when these two constraints are further alleviated, income inequality starts to decline. By contrast, reducing the intermediation cost always lead to higher income inequality, because it benefits entrepreneurs more than workers. Among entrepreneurs, it tends to benefit those more productive and wealthier entrepreneurs, who already have higher income relative to others. We also investigate the welfare implications of financial inclusion policies. We find that policies that relax different constraints tend to bring differential benefits to agents located on the wealth-productivity distribution. Although reducing the credit entry cost benefits every agent in the economy, welfare of some agents falls when the collateral constraint is relaxed or when the intermedication cost is reduced because of general equilibrium effects.

Next, we study the equilibrium interactions of these financial constraints. We conduct a sequence of counterfactual experiments in which we measure the percent growth in steady-state GDP after relaxing one financial constraint, conditional on different values of the other constraints. Confirming our theoretical insights, the simulation suggests that relaxing multiple constraints together in general equilibrium tend to amplify the effect through the intensive margin but dampen the impact through the extensive margin. Quantitatively, we find that the interaction effect on the intensive margin tends to dominate, indicating that effective financial inclusion policies should be designed to relax the most binding constraint.

Policies that are effective in generating GDP growth in one country may not be as equally effective in others. Our model provides a framework to systematically study the potential impacts of different financial inclusion policies, and hence, to identify the bottleneck constraint in a financial system. To demonstrate this idea, we calibrate our model to six representative countries, including three emerging market economies, Pakistan, Bangladesh, and Brazil, and three low-income countries, Uganda, Nepal, and Kyrgyzstan. Our model simulation indicates that these countries suffer from different financial constraints, which are also reflected in the sharply different characteristics of their financial systems.

Related Literature A growing theoretical literature has emphasized the aggregate and distributional impacts of financial intermediation in models of occupational choice and financial frictions. Banerjee and Newman (1993) develop a framework with occupation choice to capture the process of economic development. Lloyd-Ellis and Bernhardt (2000) extend the model to explain income inequality and the existence of a Kuznets curve. Cagetti and Nardi (2006) build on the framework to show that the introduction of a bequest motive generates lifetime savings profiles more consistent with data. In these studies, improved financial intermediation leads to higher entry into entrepreneurship, higher productivity and investment, and a general equilibrium effect that increases wages. Moreover, the models suggest that the distribution of wealth or the joint distribution of wealth and productivity is critical.

A related literature has found sizable impacts of improved financial intermediation on aggregate productivity and income (Gine and Townsend, 2004; Jeong and Townsend, 2007, 2008; Amaral and Quintin, 2010; Buera, Kaboski and Shin, 2011; Greenwood, Sanchez and Wang, 2013). Buera, Kaboski and Shin (2011) incorporate forward-looking agents in an occupational choice framework, and show that financial frictions account for a substantial part of the observed cross-country differences in output per worker and aggregate TFP. Moreover, Buera, Kaboski and Shin (2012) focus on the general equilibrium effects of micro finance. They find that the impact of scaling-up micro finance on per-capita income is small, because of the ensuing redistribution of income from high-savers to low-savers, but the vast majority of the population benefits from higher wages. Moll (2014) shows that the impact of financial frictions on GDP and TFP depends on the persistence of idiosyncratic shocks, and that the short-run effects of financial frictions tend to be larger than their long-run impacts.

Our model builds on this occupational choice framework, but with novel features. We focus on multiple dimensions of financial deepening within an economy. Although these dimensions have typically been considered separately in the previous literature, our paper provides a unified framework for examining them individually as well as jointly. Unlike previous studies, our model allows us to also uncover how different frictions interact with each other. In this sense, our paper is related to studies in which multiple financial frictions co-exist and are compared. Clementi and Hopenhayn (2006) and Albuquerque and Hopenhayn (2004) argue that moral hazard and limited commitment have different implications for firm dynamics. Abraham and Pavoni (2005) and Doepke and Townsend (2006) discuss how consumption allocations differ under moral hazard with and without hidden savings versus full information. Martin and Taddei (2013) study the implications of adverse selection on macroeconomic aggregates and contrast them with those under limited commitment. Karaivanov and Townsend (2014) estimate the financial/information regime in place for households (including those running businesses) in Thailand and find that a moral hazard constrained financial regime fits the data best in urban areas, while a more limited savings regime is more applicable for rural areas. Similarly, Paulson, Townsend and Karaivanov (2006) argue that moral hazard best fits the data in the more urban Central region of Thailand but not in the more rural Northeast. Kinnan (2014) uses a different metric based on the first-order conditions characterizing optimal insurance under moral hazard, limited commitment, and hidden income to distinguish between these regimes in Thai data. Finally, Moll, Townsend and Zhorin (2014) use a general equilibrium framework that encompasses different types of frictions, and examine the equilibrium interactions among various frictions. Our paper is related to these studies, but we emphasize the rich interactions among financial constraints, which in partial equilibrium can be complements on the intensive margin and substitutes on the extensive margin. Our paper also constitutes a normative policy analysis. By developing a quantitative macroeconomic framework and disciplining it with micro data, we shed light on a number of policy issues. For instance, what financial frictions are most relevant for the economy's GDP and income inequality? And what is the impact of alleviating these financial frictions individually or jointly?

Our paper is also broadly related to the literature on misallocation (Hsieh and Klenow, 2009; Caselli and Gennaioli, 2013; Midrigan and Xu, 2014; Moll, 2014) and inequality (Davies, 1982; Huggett, 1996; Aghion and Bolton, 1997; Castaneda, Diaz-Gimenez and Rios-Rull, 2003; Nardi, 2004). Our contribution is to show that policy options that target different financial sector frictions have different impacts on resource allocation and inequality.

The remainder of the paper is organized as follows. The next section provides sets out the structure of the model. Section 3 discusses the differential impacts of relaxing different financial constraints and their interactions. Section 4 presents calibration and quantitative results. Finally, Section 5 concludes.

2 Model

We develop a model with individual-specific technologies and imperfect financial markets to study the equilibrium interactions among different financial frictions and their implications on GDP, TFP, and inequality.

2.1 Agents

Heterogeneity and Demographics Time is discrete and indexed by t. The economy is populated by a continuum of agents of measure one and there is no population growth. Agents live indefinitely and are heterogeneous in terms of wealth b_t and entrepreneurial productivity z_t. Wealth evolves endogenously, determined by forward-looking saving decisions. Productivity z_t follows an exogenous stochastic process. In particular, with probability γ, agents retain their productivity in the previous period; with probability $1-\gamma$, agents draw a new productivity from a time-invariant Pareto distribution $\mu(z)$ governed by the tail parameter θ. The shocks to productivity can be interpreted as changes in market conditions that affect the profitability of individual skills (Buera, Kaboski and Shin, 2011).

We denote $h_t(b,z)$ as the Probability Density Function (PDF) for the joint distribution of ability and wealth at the beginning of period t. Denote the set of agents as $\Phi_t = \{(b,z) : h_t(b,z) > 0\}$.

Preferences Agents derive utility from consumption and have preferences

$$\mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \frac{c_s^{1-\sigma}}{1-\sigma} \right], \tag{2.1}$$

where β is the discount factor and σ is the risk aversion parameter.

Technology In any period t, agents can make occupation choices between workers and entrepreneurs. Each worker supplies one unit of labor inelasticly and earns the equilibrium wage w_t. Each entrepreneur operates a technology that uses capital k_t and labor l_t as inputs to produce output. The profitability of an entrepreneurial business depends on the agent's productivity, specifically, the output $f(k_t, l_t, z_t)$ is given by

$$f(k_t, l_t, z_t) = z_t (k_t^{\alpha} l_t^{1-\alpha})^{1-\nu}, \tag{2.2}$$

where $1-\nu$ is the span-of-control parameter, representing the share of output accruing to the variable factors. Out of this, a fraction α goes to capital, and $1-\alpha$ goes to labor. Production exhibits diminishing returns to scale, with $\nu > 0$. Capital depreciates at rate δ.

2.2 Financial Markets

The only asset in the economy is productive capital. This is equivalent to assuming that there is a perfect technology that can freely transform capital into consumption goods. There is a perfectly competitive financial intermediary that receives deposits from all agents and lends to entrepreneurs. The deposit interest rate is r_t, determined endogenously by the capital market clearing condition. In this paper, we focus on within-period credit, or intra-period loans, for production purposes. We do not allow borrowing for consumption smoothing across periods by imposing $b_t \geq 0$. Therefore, only entrepreneurs can borrow from intermediaries.

The key distinction of our model from existing models is that we introduce multiple financial frictions to financial markets. These are described below in turn.

Financial Access Entrepreneurs need to incur an upfront fixed credit entry cost, ψ units of consumption goods, to borrow from intermediaries. The modeling of fixed entry cost for obtaining credit has also been adopted by Greenwood and Jovanovic (1990) and Townsend and Ueda (2006) among others. It captures, for example, various fees associated with financial accounts, the cost of bookkeeping, exchange (transportation), etc (Townsend, 1983*a*,*b*).

We assume that an agent lives in a "credit regime" if the agent pays the cost ψ and can borrow; that an agent lives in a "savings regime", if the agent does not pay ψ and can thereby only save. In equilibrium, the fixed entry cost ψ is more likely to exclude poor entrepreneurs from financial markets, because this amounts to a larger fraction of their initial wealth.

Financial Depth The credit contract that entrepreneurs obtain is subject to a collateral requirement due to limited enforceability of debt contracts. In particular, consider an entrepreneur with wealth b_t who comes to financial intermediaries for a loan x_t at the competitive equilibrium lending interest rate r_t^l. After obtaining the loan x_t, the entrepreneur transforms the wealth-on-hand $b_t - \psi + x_t$ costlessly into capital $k_t = b_t - \psi + x_t$. The physical capital k_t is then used as collateral to secure the loan x_t. The entrepreneur is free to default and walk away with her income and wealth. The only punishment is that the bank seizes the collateral. Following Jermann and Quadrini (2012), we assume that the liquidation value of physical capital is uncertain at the moment of contracting. With probability $1-\xi$, intermediaries recover the full value k_t, but with probability ξ, the recovery value is zero. Thus to avoid default, the amount of bank loan x_t that intermediaries are willing to lend is restricted by

$$x_t \leq (1-\xi)k_t. \tag{2.3}$$

Substituting x_t, we can derive the entrepreneur's productive capital constraint in terms of her wealth b_t:

$$k_t \leq \frac{b_t - \psi}{\xi} \tag{2.4}$$

The single parameter $\xi \in [0,1]$ parsimoniously captures the tightness of the borrowing constraint, with $\xi = 1$ corresponds to financial autarky, where all capital must be self-financed by entrepreneurs.

Intermediation Efficiency We assume that the lending interest rate r_t^l is higher than the deposit interest rate r_t by a margin χ, i.e. $r_t^l = r_t + \chi$. Thus for a loan x_t, entrepreneurs pay intermediation fees equal to χx_t units of consumption goods. The interest rate spread χ reflects the efficiency of financial intermediation. For example, as noted by Townsend (1983*a*,*b*), costly intermediation could arise from the cost of enforcement and monitoring when there is imperfect information. In our previous working paper (Dabla-Norris et al., 2015), this intermediation cost is micro founded by costly state verification (Townsend, 1979).

The distinguishing feature of our model is that three sources of financial frictions, denoted by $\Omega = (\psi, \xi, \chi)$, are examined in a unified framework. Each friction reflects one particular aspect of financial market imperfection.[3] Modelling these frictions in a unified framework allows us to study the equilibrium interactions among different financial frictions. Moreover, by calibrating the model using micro data, we can shed light on the tightness of each constraint and identify the most binding friction in an economy.

[3] Existing studies have emphasized the quantitative importance of each financial friction in explaining cross-country economic growth patterns. For example, Greenwood and Jovanovic (1990); Townsend and Ueda (2006) focus on the credit entry cost. Buera, Kaboski and Shin (2011); Buera and Shin (2013); Moll (2014) focus on the collateral constraint. Greenwood, Sanchez and Wang (2010, 2013) quantifies the importance of intermediation cost.

2.3 Agents' Problem

We formulate the agent's problem recursively. Denote $V_t(z_t, b_t)$ as the value function of the agent of type (z_t, b_t). Denote $W_t(z_t, b_t)$ and $E_t(z_t, b_t)$ as the agent's value if she chooses to be a worker or an entrepreneur in period t. Occupation choice is made to maximize utility,

$$V_t(z_t, b_t) = \max \{W_t(z_t, b_t), E_t(z_t, b_t)\}. \tag{2.5}$$

The worker's value is given by

$$\begin{aligned} W_t(z_t, b_t) &= \max_{c_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}(z_{t+1}, b_{t+1})\right] \\ &\text{subject to} \\ c_t + b_{t+1} &= (1+r_t)b_t + w_t, \\ c_t, b_{t+1} &\geq 0, \end{aligned} \tag{2.6}$$

The entrepreneur decides whether to borrow from financial intermediaries. Denote $E_t^s(z_t, b_t)$ and $E_t^c(z_t, b_t)$ as the entrepreneur's value when productive capital is financed with savings only or with credit. The borrowing decision is made to maximize utility,

$$E_t(z_t, b_t) = \max \{E_t^s(z_t, b_t), E_t^c(z_t, b_t)\}. \tag{2.7}$$

In the savings regime, the agent self finances production and the remaining wealth is deposited in financial intermediaries to receive interest earnings. The value function in the savings regime is

$$\begin{aligned} E_t^s(z_t, b_t) &= \max_{c_t, k_t, l_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}(z_{t+1}, b_{t+1})\right] \\ &\text{subject to} \\ c_t + b_{t+1} &= z_t (k_t^\alpha l_t^{1-\alpha})^{1-\nu} + (1-\delta)k_t - w_t l_t + (1+r_t)(b_t - k_t), \\ k_t &\leq b_t, \\ c_t, k_t, l_t, b_{t+1} &\geq 0, \end{aligned} \tag{2.8}$$

In the credit regime, the agent takes loans to finance production. The amount of wealth that can be used as collateral is $b_t - \psi$ after paying the credit entry cost. The interest rate on loans is higher than the equilibrium deposit rate by a margin χ. Without loss of generality, we only consider the case in which the agent invests k_t strictly higher than b_t in the credit regime, because investing $k_t \leq b_t$ but paying the credit entry cost is obviously not optimal. Thus we can write the value function in the credit regime as

$$E_t^c(z_t, b_t) = \max_{c_t, k_t, l_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}(z_{t+1}, b_{t+1}) \right] \tag{2.9}$$

subject to

$$c_t + b_{t+1} = z_t (k_t^{\alpha} l_t^{1-\alpha})^{1-\nu} + (1-\delta) k_t - w_t l_t - (1 + r_t + \chi)(k_t - b_t + \psi),$$

$$\xi k_t \leq b_t - \psi,$$

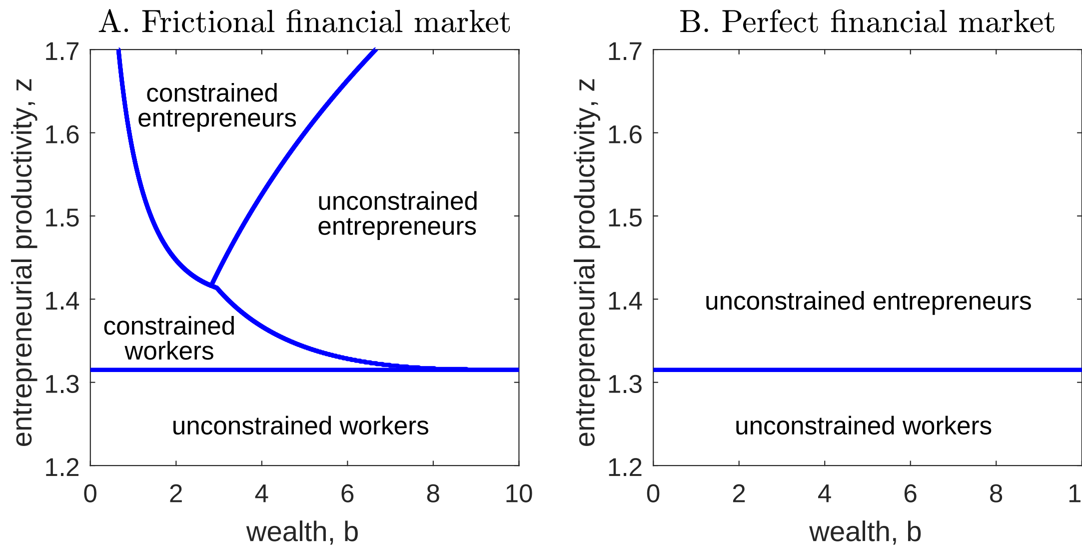
$$c_t, l_t, b_{t+1} \geq 0 \text{ and } k_t > b_t,$$

Panel A of Figure 2 illustrates the choice of occupation for agents of different types (b, z). Depending on whether occupation choice is constrained by wealth, we identify four categories of agents, separated by the solid lines: unconstrained workers, constrained workers, constrained entrepreneurs, and unconstrained entrepreneurs. As shown in the figure, there is a certain threshold level of talent (1.32) below which agents always find working for a wage better than operating a business. These agents are identified as unconstrained workers, because their productivity is so low that they never find it optimal to become entrepreneurs. Above this level of productivity, there are three regions. The left region refers to constrained workers. These agents are productive but do not have sufficient wealth to operate businesses at a profitable scale. The middle region represents constrained entrepreneurs who have sufficient wealth to operate profitable businesses, but the scale of businesses is still constrained by wealth. Agents in the right region choose to be entrepreneurs, operating businesses at the unconstrained scale, with the marginal return on capital equal to the marginal cost of production. Thus, they are identified as unconstrained entrepreneurs.

In the presence of financial market frictions, the decision to become an entrepreneur and the scale of business not only depend on productivity z_t but also on wealth b_t. By contrast, when the financial market is perfect, i.e. $\psi = \chi = \xi = 0$. Panel B of Figure 2 indicates that productive agents always choose to be entrepreneurs and operate their business at the unconstrained scale. We formally state this result in Proposition 1.

Proposition 1. *In the absence of financial frictions, there exists a threshold of entrepreneurial productivity $\underline{z}_t$, the agent chooses to be an entrepreneur if and only if $z \geq \underline{z}_t$.*

Proof. See Appendix B.1. □



Note: Panel A plots the occupation choice map in the presence of financial frictions ($\psi = 0.05$, $\chi = 0.05$, $\xi = 0.5$). Panel B plots the occupation choice map in a perfect financial market ($\psi = \chi = \xi = 0$) in partial equilibrium with the same interest rate and wage as Panel B (the other parameters are: $r = 0.05$, $w = 0.8$, $\delta = 0.06$, $\nu = 0.21$, $\alpha = 0.33$). In general equilibrium, when we move from an economy with financial frictions to an economy with a perfect financial market, the interest rate and wage will also change, which we analyze in Section 4.

Figure 2: The occupation choice map with/without financial frictions.

2.4 Competitive Equilibrium

Given an initial joint probability density distribution of wealth and talent $h_0(b,z)$, a competitive equilibrium consists of allocations $\{c_t(b,z), k_t(b,z), l_t(b,z)\}_{t=0}^{\infty}$, sequences of joint distributions of wealth and talent $\{h_t(b,z)\}_{t=0}^{\infty}$ and prices $\{r_t, w_t\}_{t=0}^{\infty}$, such that:

(1). Agents of type (b,z) optimally choose the underlying regime, occupations, consumption $c_t(b,z)$, capital $k_t(b,z)$, and labor $l_t(b,z)$ to maximize utility at $t \geq 0$.

(2). The capital market clears at all $t \geq 0$,

$$\iint_{(b,z)\in\Phi_t^C} [k_t(b,z) - b + \psi] h_t(b,z) \mathrm{d}b\mathrm{d}z = \iint_{(b,z)\notin\Phi_t^E} b h_t(b,z) \mathrm{d}b\mathrm{d}z + \iint_{(b,z)\in\Phi_t^S} [b - k_t(b,z)] h_t(b,z) \mathrm{d}b\mathrm{d}z, \tag{2.10}$$

where Φ_t^E is the set of agents who choose to be entrepreneurs at time t; Φ_t^S and Φ_t^C are the sets of entrepreneurs in the savings and credit regimes. We have $\Phi_t^E = \Phi_t^S \cup \Phi_t^C$.

(3). The labor market clears at all $t \geq 0$,

$$\iint_{(b,z)\in\Phi_t^E} l_t(b,z) h_t(b,z) \mathrm{d}b\mathrm{d}z = \iint_{(b,z)\notin\Phi_t^E} h_t(b,z) \mathrm{d}b\mathrm{d}z. \tag{2.11}$$

(4). $\{h_t(b,z)\}_{t=1}^{\infty}$ evolves according to the equilibrium mapping:

$$h_{t+1}(b',z')\mathrm{d}b\mathrm{d}z = \gamma dz \int_{(b,z')\in\Phi_t} \mathbb{1}_{\{b^*=b'\}} h_t(b,z')\mathrm{d}b + (1-\gamma)\mu(z')dz \iint_{(b,z)\in\Phi_t} \mathbb{1}_{\{b^*=b'\}} h_t(b,z)\mathrm{d}b\mathrm{d}z, \tag{2.12}$$

where b^* is the wealth at $t+1$ implied by the optimal policy functions of agents (b,z). $\mathbb{1}_{b^*=b'}$ is an indicator function which equals 1 if $b^* = b'$, and equals 0 otherwise. The left hand side of equation (2.12) is the probability mass of agents with (b',z') at $t+1$. The right hand side sums up the transition probability to (b',z') from any arbitrary (b,z) at t. With probability γ, the agent keeps the current talent z' and transits to b' from (b,z') if $b^* = b'$. With probability $1-\gamma$, the agent draws a new talent, which equals z' with probability $\mu(z')dz$.

The steady state of the economy is defined as the invariant joint distribution of wealth and talent $h(b,z)$,

$$h(b,z) = \lim_{t\to\infty} h_t(b,z). \tag{2.13}$$

For expositional purposes, we denote the steady-state interest rate and wage as r and w, and the optimal policy functions as $c(b,z)$, $k(b,z)$, and $l(b,z)$ for the agent of (b,z). We denote the steady-state sets of agents, entrepreneurs, and entrepreneurs in the savings and credit regimes as Φ, Φ^E, Φ^S, Φ^C.

2.5 Computing GDP in The Steady State

In the steady state, the income $I(b,z)$ for each agent of (b,z) is equal to the sum of her wage, interest, and (business) profit, if any. The amount of each component depends on her occupation and credit access. In particular,

$$I(b,z;\Omega) = \tag{2.14}$$

$$\begin{cases} w + rb, & \text{workers;} \\ z(k^{\alpha}l^{1-\alpha})^{1-\nu} - \delta k - wl + r(b-k), & \text{entrepreneurs in the savings regime;} \\ z(k^{\alpha}l^{1-\alpha})^{1-\nu} - \delta k - wl - \psi - (r+\chi)(k-b+\psi), & \text{entrepreneurs in the credit regime,} \end{cases}$$

where k and l are short notations for $k(b,z;\Omega)$ and $l(b,z;\Omega)$.

According to the income approach, the economy's steady-state GDP is equal to total national income plus depreciation

$$\text{GDP}_\Omega = \iint\limits_{(b,z)\in\Phi} I(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \iint\limits_{(b,z)\in\Phi^E} \delta k(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \tag{2.15}$$

In Appendix A, we show that equation (2.15) can be rewritten as

$$\text{GDP}_\Omega = \iint\limits_{(b,z)\in\Phi^S} y^S(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \iint\limits_{(b,z)\in\Phi^C} y^C(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \tag{2.16}$$

where $y^S(b,z;\Omega)$ and $y^C(b,z;\Omega)$ are entrepreneurs' output net of the deadweight loss arising from financial frictions in the savings and credit regimes:

$$y^S(b,z;\Omega) = z\left[k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha}\right]^{1-\nu}, \tag{2.17}$$

$$y^C(b,z;\Omega) = z\left[k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha}\right]^{1-\nu} - \chi\left[k(b,z;\Omega) - b + \psi\right] - \psi. \tag{2.18}$$

3 Distinguishing the Impact of Financial Constraints

In our model, financial deepening and inclusion are reflected by lower values of the parameters, $\Omega = (\psi, \xi, \chi)$. In this section, we explore the impact of relaxing these financial constraints separately and in combination. We first analyze individuals' output and credit access conditions in partial equilibrium to uncover the model mechanisms and distinguish the impact of different financial constraints. We then decompose GDP and TFP to shed light on the macroeconomic effects of financial inclusion policies.

3.1 Implications in Partial Equilibrium

We distinguish the effect of financial inclusion on the extensive and the intensive margins.[4] On the one hand, relaxing financial constraints can increase GDP through the extensive margin by increasing the credit access ratio (i.e. moving entrepreneurs from the savings regime to the credit regime). On the other hand, relaxing financial constraints enables entrepreneurs in the credit regime to produce more output, boosting GDP through the intensive margin. Consider the steady state of the economy Ω. To illustrate the

[4]Our definition of the two margins are different from Banerjee and Moll (2010)'s, who focus on capital misallocation. Banerjee and Moll (2010) define misallocation at the intensive margin as the difference in marginal products of capital among capital users and the misallocation at the extensive margin as talented people who never have enough funds to set up a business.

mechanism, we first focus on the partial equilibrium with fixed interest rate r and wage w. We consider constant returns to scale production function (i.e. $\nu = 0$) to simplify the algebra.

Extensive Margin Financial liberalization increases GDP by allowing more entrepreneurs to enter the credit regime. Denote $\underline{b}(z;\Omega)$ as the steady-state threshold of wealth above which entrepreneurs with productivity z choose to access credit. A lower $\underline{b}(z;\Omega)$ implies that, all else equal, entrepreneurs are more likely to enter the credit regime. In Proposition 2, we show that relaxing financial constraints Ω (decreasing ψ, ξ, or χ) lowers agents' wealth thresholds $\underline{b}(z;\Omega)$, reflecting the effect of policies through the extensive margin.

Proposition 2. *In the credit regime with fixed interest rates r and wages w, and when $\nu = 0$:*

i. Tightening (relaxing) each financial constraint reduces (improves) credit access:

$$\frac{\partial \underline{b}}{\partial \psi} \geq 0, \quad \frac{\partial \underline{b}}{\partial \xi} \geq 0, \quad \frac{\partial \underline{b}}{\partial \chi} \geq 0. \tag{3.1}$$

ii. $\underline{b}(z;\Omega)$ has submodularity in Ω, suggesting that the effect of relaxing one constraint on the extensive margin is smaller when the other constraints are more relaxed:

$$\frac{\partial^2 \underline{b}}{\partial \psi \partial \xi} = \frac{\partial^2 \underline{b}}{\partial \xi \partial \psi} \geq 0, \quad \frac{\partial^2 \underline{b}}{\partial \xi \partial \chi} = \frac{\partial^2 \underline{b}}{\partial \chi \partial \xi} \geq 0, \quad \frac{\partial^2 \underline{b}}{\partial \chi \partial \psi} = \frac{\partial^2 \underline{b}}{\partial \psi \partial \chi} \geq 0. \tag{3.2}$$

Proof. See Appendix B.2. □

Proposition 2.i indicates that relaxing any constraint can unambiguously reduce the wealth threshold $\underline{b}(z;\Omega)$, which facilitates credit access. The underlying mechanisms for relaxing different constraints are not identical. Lowering the credit entry cost ψ induces entrepreneurs to enter the credit regime by decreasing the ex-ante cost of borrowing, while a lower intermediation cost χ reduces the ex-post cost of borrowing. Relaxing the borrowing constraint ξ motivates entrepreneurs to obtain credit by increasing their profits in the credit regime.

Importantly, Theorem 2.ii indicates that the three financial constraints dampen each other's effect on the extensive margin. For example, when the credit participation cost is lower, relaxing the borrowing constraint or reducing the intermediation cost have a smaller effect on reducing $\underline{b}(z;\Omega)$. This is because a lower credit entry cost results in a lower wealth threshold $\underline{b}(z;\Omega)$, thus relaxing the borrowing constraint and reducing the intermediation cost have less of an impact on further reducing this threshold. In other words, when the credit entry cost is low, the credit access ratio is already high. As such, naturally there is little room for increasing access further through the other two channels.

Intensive Margin Financial deepening also increases GDP by increasing the output of entrepreneurs in the credit regime. In Proposition 3, we state the impact of relaxing financial constraints on entrepreneurs' output (defined in equation 2.17) in the credit regime.

Proposition 3. *In the credit regime with fixed interest rates r and wages w, and when $\nu = 0$:*

i. Tightening (relaxing) each financial constraint reduces (raises) output:

$$\frac{\partial y^C}{\partial \psi} \leq 0, \quad \frac{\partial y^C}{\partial \xi} \leq 0, \quad \frac{\partial y^C}{\partial \chi} \leq 0. \tag{3.3}$$

ii. $y^C(b, z; \Omega)$ has supermodularity, suggesting that the effect of relaxing one constraint on the intensive margin is larger when the other constraints are more relaxed:

$$\frac{\partial^2 y^C}{\partial \psi \partial \xi} = \frac{\partial^2 y^C}{\partial \xi \partial \psi} \geq 0, \quad \frac{\partial^2 y^C}{\partial \xi \partial \chi} = \frac{\partial^2 y^C}{\partial \chi \partial \xi} \geq 0, \quad \frac{\partial^2 y^C}{\partial \chi \partial \psi} = \frac{\partial^2 y^C}{\partial \psi \partial \chi} \geq 0. \tag{3.4}$$

Proof. See Appendix B.3. □

Proposition 3.i indicates that relaxing any constraint increases entrepreneurs' output in the credit regime. Proposition 3.ii says that relaxing any two constraints has complementary effects on output. For example, when the credit entry cost is low, entrepreneurs are left with more wealth after entering the credit regime. Since both the amount of credit and the total intermediation cost are proportional to wealth, relaxing the collateral constraint or reducing the intermediation cost would have a larger effect on output.

Our partial equilibrium analyses reveal that different constraints are complements on the intensive margin and substitutes on the extensive margin. Therefore, understanding how these constraints interact in equilibrium is useful for designing more effective policies. We study these issues in Section 4.4.

3.2 Aggregate and General Equilibrium Implications

We now discuss the general equilibrium impact of financial constraints on the aggregate economy.

Proposition 4. *Consider the financial liberalization that relaxes the constraints from $\Omega = (\psi, \xi, \chi)$ to $\Omega' = (\psi', \xi', \chi')$ with $\Omega' \leq \Omega$, the increase in steady-state GDP can be decomposed* *into three margins:*

$$\begin{aligned} \mathrm{GDP}_{\Omega'} - \mathrm{GDP}_{\Omega} = & \int_z \int_{\underline{b}(z;\Omega')}^{\underline{b}(z;\Omega)} \left[y^C(b,z;\Omega')h(b,z;\Omega') - y^S(b,z;\Omega)h(b,z;\Omega) \right] dbdz \\ & + \int_z \int_{\underline{b}(z;\Omega)}^{\infty} \left[y^C(b,z;\Omega')h(b,z;\Omega') - y^C(b,z;\Omega)h(b,z;\Omega) \right] dbdz \\ & + \int_z \int_{0}^{\underline{b}(z;\Omega')} \left[y^S(b,z;\Omega')h(b,z;\Omega') - y^S(b,z;\Omega)h(b,z;\Omega) \right] dbdz. \end{aligned} \tag{3.5}$$

Proof. See Appendix B.4. □

The first term in the RHS of equation (3.5) captures the gains on the extensive margin as more entrepreneurs move from the savings regime to the credit regime, and produce more, i.e. $\underline{b}(z;\Omega') < \underline{b}(z;\Omega)$. The second term captures the gains on the intensive margin, i.e. $y^C(b,z;\Omega') > y^C(b,z;\Omega)$ for the entrepreneurs who are already in the credit regime and remain in this regime after financial deepening. The two terms also implicitly include the general equilibrium effect as the wage and interest rate increase after financial liberalization. The third term reflects the pure general equilibrium effect on the entrepreneurs who remain in the savings regime after financial liberalization.

In general, financial deepening benefits entrepreneurs either by allowing them to access credit at lower cost (lower ψ and χ) or to access more credit (lower ξ), both of which increase entrepreneurs' profits and demand for capital and labor. Higher demand boosts the equilibrium wage and interest rate, which benefits workers as they supply labor and make deposit. However, less productive and wealth constrained entrepreneurs may lose if they remain in the savings regime after financial liberalization. Intuitively, these entrepreneurs do not benefit from the improved access to credit but suffer from the higher wage payments. Whether financial inclusion leads to higher or lower income inequality depends on the joint distribution of wealth and talent as well as which financial constraint is relaxed. We study the effect of financial inclusion on income inequality and welfare in Section 4.2 and 4.3.

We now turn to the analysis of the economy's TFP. Following Buera and Shin (2013), we define the model-implied steady-state TFP as

$$\mathrm{TFP}_{\Omega} = \frac{Y_{\Omega}}{K_{\Omega}^{\alpha} L_{\Omega}^{1-\alpha}}, \tag{3.6}$$

where Y_{Ω}, K_{Ω}, and L_{Ω} are steady-state aggregate output, capital, and labor defined in Appendix B.5. Moll (2014) shows that when entrepreneurs have constant-return-to-scale production technology, TFP defined by equation (3.6) is equal to average individual productivity weighted by wealth.

By exploring the equivalence between growth accounting by factors and growth accounting by regimes, we decompose the economy's change in steady-state TFP along similar dimensions.[5]

Proposition 5. *Consider the financial liberalization that relaxes the constraints from $\Omega = (\psi, \xi, \chi)$ to $\Omega' = (\psi', \xi', \chi')$ with $\Omega' \leq \Omega$, the increase in steady-state TFP can be decomposed into three margins:*

$$\begin{aligned}\mathrm{TFP}_{\Omega'} - \mathrm{TFP}_{\Omega} = & g_{p^c}\left[s_y^c - s_y^s - \alpha(s_k^c - s_k^s) - (1-\alpha)(s_l^c - s_l^s)\right]\mathrm{TFP}_{\Omega} \\ & + \left[s_y^c g_{\overline{y}^c} - \alpha s_k^c g_{\overline{k}^c} - (1-\alpha) s_l^c g_{\overline{l}^c}\right]\mathrm{TFP}_{\Omega} \\ & + \left[s_y^s g_{\overline{y}^s} - \alpha s_k^s g_{\overline{k}^s} - (1-\alpha) s_l^s g_{\overline{l}^s}\right]\mathrm{TFP}_{\Omega}, \end{aligned} \tag{3.7}$$

where s_y^s, s_y^c, s_k^s, s_k^c, s_l^s, s_l^c are the average fraction of output, capital, and labor associated with entrepreneurs in the savings regime and the credit regime. g_{p^c} is the percent change in the fraction of entrepreneurs in the credit regime after financial liberalization. $g_{\overline{y}^s}$, $g_{\overline{y}^c}$, $g_{\overline{k}^s}$, $g_{\overline{k}^c}$, $g_{\overline{l}^s}$, and $g_{\overline{l}^c}$ are the percent change in the average output, capital, and labor in the savings regime and the credit regime after financial deepening.

Proof. See Appendix B.5. □

In equation (3.7), the first term captures TFP growth through the extensive margin, attributed to the entrepreneurs who move from the savings regime to the credit regime. The second term captures TFP growth through the intensive margin, attributed to the entrepreneurs who remain in the credit regime before and after financial deepening. The third term captures the general equilibrium effect from entrepreneurs who remain in the savings regime after financial deepening.

4 Quantitative Analyses

In this section, we conduct quantitative analyses to systematically examine different financial inclusion policies in general equilibrium. Using the calibrated model, we first evaluate the aggregate implications of financial inclusion and shed light on the distributional welfare implications. We then study the equilibrium interactions among the

[5] Our decomposition is similar to the approach of Jeong and Townsend (2007), who decompose TFP growth into the occupational-shift effect, financial-deepening effect, capital-heterogeneity effect, and sectoral-Solow-residuals.

three financial constraints. Finally, we calibrate the model to six countries to illustrate the model's ability to match a wide range of economies exhibiting different financial system characteristics and to identify the most binding financial constraint.

4.1 Data and Calibration

We use firm-level data from the World Bank Enterprise Survey and macroeconomic indicators from the World Development Indicators to discipline the model. The World Bank Enterprise Survey is a firm-level survey of a representative sample of an economy's private sector.[6]

We calibrate the model's parameters by matching the moments for the Phillippines. The sample contains 1335 firms interviewed from November 2014 through May 2016. We take the Phillippines as our benchmark calibration because the financial system in the Phillippines is moderately developed and there is room for further improvement. In Section 4.5, we calibrate the model to target moments for other countries and show the model's ability to conduct cross-country analyses.[7]

We set the risk aversion parameter $\sigma = 1.5$ following standard practice. The one-year depreciation rate δ is set to be 0.06. The aggregate income share of capital α is set to be 0.33.

The remaining 7 parameters ψ, ξ, χ, β, ν, θ, γ are calibrated by matching 10 moments in the data (see Table 1), denoted by $\{m_i\}_{i=1}^{10}$. We choose those moments that are informative about these parameters. Because changing each parameter affects all the moments simultaneously, albeit to a different extent, we have to jointly calibrate these parameters to ensure that all the moments are well matched. Specifically, we start with an arbitrary distribution of agents in terms of wealth and productivity and simulate their behavior according to the computed policy functions. We choose the simulation period to be sufficiently long so as to ensure that the model economy reaches the steady state. Based on the stationary invariant distribution, we compute the model-implied moments, $\{\hat{m}_i\}_{i=1}^{10}$, and adjust the parameters to minimize

$$L = \sum_{i=1}^{10} \left| \frac{\hat{m}_i - m_i}{m_i} \right|. \tag{4.1}$$

[6]The surveys cover a broad range of topics including access to finance, corruption, infrastructure, crime, competition, and performance measures. To date, the surveys have been conducted in 139 countries with the goal of producing research on the microeconomic foundations of growth.

[7]A previous version of our model (Dabla-Norris et al., 2014) has been applied to analyzing policy issues in various countries in Africa and Latin America by the IMF, the World Bank, and the IDB.

Table 1: Calibration and moments in data and model.

	Data	Model	Parameters
Firms with credit (%)	29.9	29.5	$\psi = 0.95$
Collateral (% of loans)	156.7	156.4	$\xi = 0.33$
Interest rate spread (%)	4.0	4.0	$\chi = 0.04$
Real interest rate (%)	6.2	6.8	$\beta = 0.88$
Income GINI coefficient	0.40	0.41	$\nu = 0.19$
Top 5% employment share (%)	43.7	43.7	$\theta = 3.4$
Top 10% employment share (%)	57.6	58.6	$\sigma = 1.5$
Top 20% employment share (%)	73.2	71.5	$\delta = 0.06$
Top 40% employment share (%)	88.8	85.3	$\alpha = 0.33$
Gross savings (% of GDP)	44.2	41.2	$\gamma = 0.89$

Below we briefly discuss the moments used in our calibration. The three key parameters governing the financial frictions are calibrated to match the credit access ratio (i.e. the percent of firms with access to credit), the collateral-to-loan ratio, and the interest rate spread. The interest rate spread directly identifies the intermediation efficiency parameter χ. For firms with access to credit, the average collateral as a percent of loans is 156.7%, indicating that for each unit of credit, the firm, on average, has to post 1.57 unit of collateral. This moment provides information about the tightness of the collateral constraint, governed by the parameter ξ. In the Philippines, the fraction of firms with credit access is 29.9%, which identifies the credit entry cost ψ.

A key parameter that determines the long-run welfare implication of financial frictions is the persistence of entrepreneurial productivity, γ. Buera and Shin (2011) and Moll (2014) show that when productivity is persistent, entrepreneurs can accumulate wealth on their own and overcome financial constraints. We thus identify the parameter γ by matching the gross savings rate as a percent of GDP.

Following the standard practice, we set the discount factor β to match the real interest rate (6.2% in 2015). The span-of-control parameter ν governs the degree of decreasing return to scale, which is informative about the profitability of entrepreneurs' business. Intuitively, a higher ν implies that entrepreneurial projects cannot be operated at larger scale, reducing the income difference between workers and entrepreneurs. We thus calibrate ν to match the income GINI coefficient in 2015 (0.4).

The parameter θ governs the distribution of agents' entrepreneurial productivity. A lower θ implies that the distribution is more right skewed, and hence top entrepreneurs will employ a larger fraction of labor in the economy. We thus calibrate θ to match the employment share distribution. Specifically, we use four brackets of employment shares, corresponding to the fraction of labor employed by top 5%, 10%, 20%, and 40% entrepreneurs ranked in terms of their business revenue.

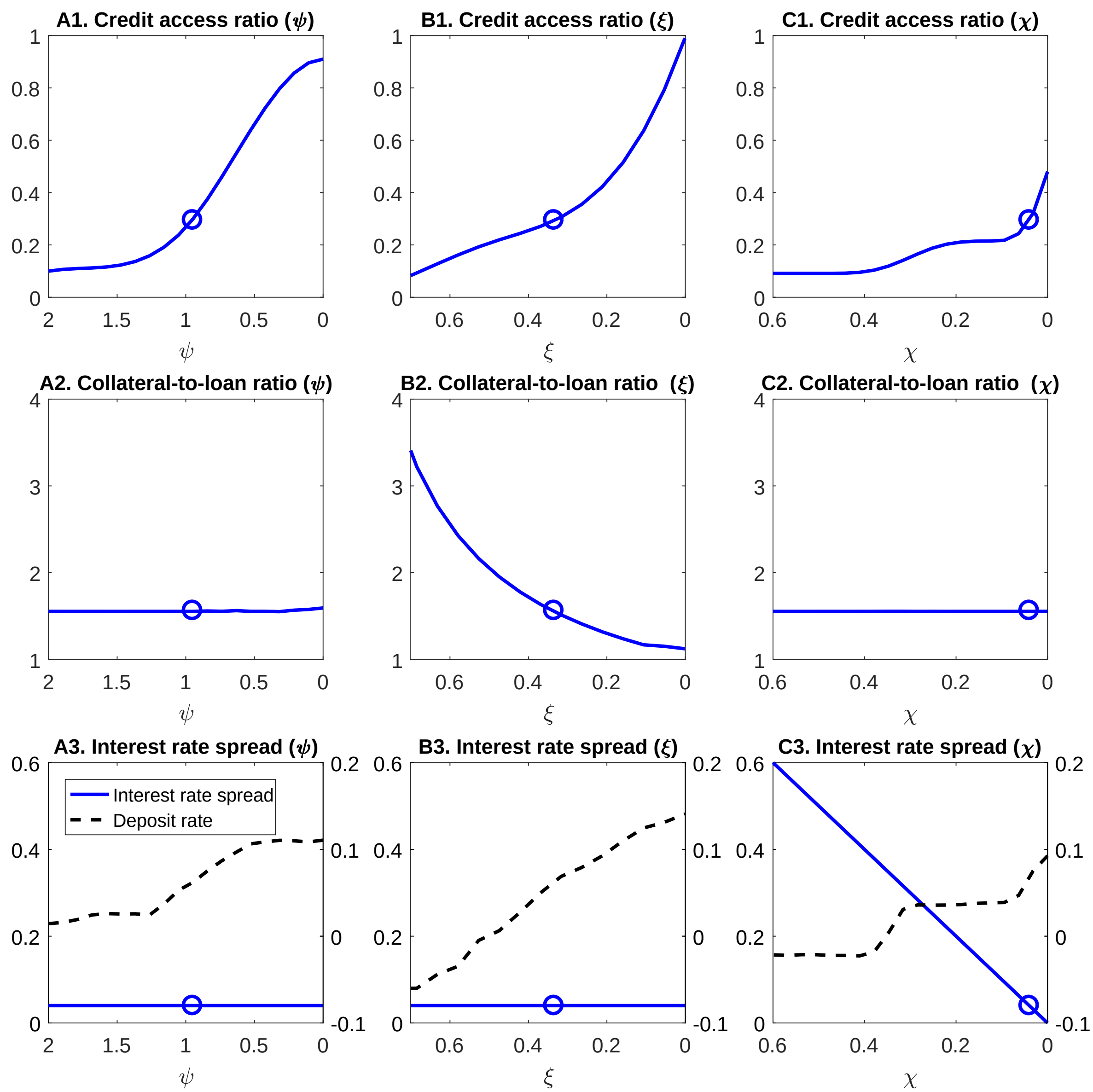
4.2 Aggregate Implications of Financial Liberalization

In this section, we analyze the implications of financial inclusion across the three dimensions of a financial system both in steady states and during transitions.

Steady States We start by examining the steady states of the Phillippines when the parameters ψ, ξ, and χ change separately, holding the other parameters at the calibrated values. Figure 3 presents the effect of these policies on the Phillippines. As shown in Panels A1, B1, C1, reducing the credit entry cost ψ, relaxing the collateral constraint ξ, and reducing the intermediation cost χ all lead to a higher credit access ratio and more inclusion. However, as we discuss in Section 2, these financial frictions are meant to capture different types of constraints. Particularly, the parameter ψ captures the ex-ante friction in obtaining credit access, but not the terms of the loan contract within the credit regime. Thus, a lower ψ increases the credit access ratio but has a negligible effect on the collateral-to-loan ratio and the interest rate spread. The other two parameters, ξ and χ, capture ex-post frictions within the credit regime. Thus, a lower ξ and a lower χ imply a lower collateral-to-loan ratio (Panel B2) and a lower interest rate spread (Panel C3). Better terms of the loan contract attract entrepreneurs to access credit by paying the credit entry cost ψ. Thus even if ψ remains unchanged, the credit access ratio increases due to lower ξ and χ.

In Figures 4, we present the simulation results on GDP, TFP and income inequality. Panels A1-C1 and A2-C2 of Figure 4 compare the change in GDP (blue solid lines in Panels A1, B1, C1) and TFP (blue solid lines in Panels A2, B2, C2) when financial deepening operates through alleviating different frictions. Panel A1 shows that when the credit entry cost ψ falls from 2 to 0, GDP increases by about 20.2% and TFP by 9.4%; Panel B1 shows that GDP and TFP increase by about 71.8% and 53.3% when the collateral constraint parameter ξ falls from 0.7 to 0; and Panel C1 shows that GDP and TFP increase by about 9.5% and 5.8% respectively, when the intermediation cost χ is lowered from 0.6 to 0. These comparative statics clearly illustrate that the sensitivity of the Phillippines' steady-state GDP and TFP with respect to each financial constraint is different. In Section 4.5, we apply our model to different countries and show the model's ability to identify the financial constraint that constrains an economy's aggregate output and productivity.

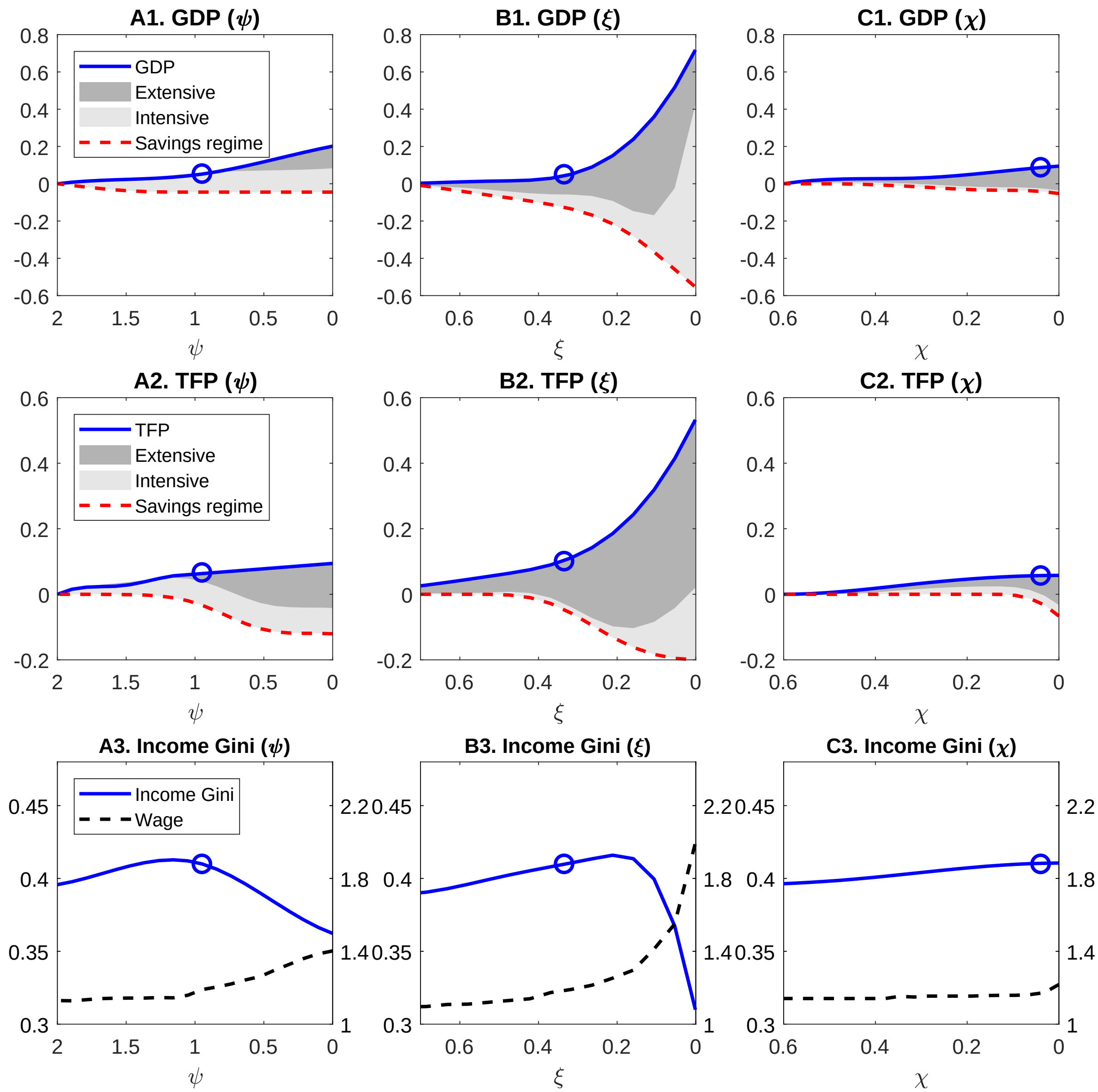
We continue to use the formulas (3.5) and (3.7) to understand the margins through which GDP and TFP increase when the three financial constraints are relaxed. Aggregate output and productivity in the savings regime all decrease, negatively contributing to the overall increase in GDP and TFP. Quantitatively, the savings regime contributes about -4.5%, -55.4%, -5.3% to GDP growth, and -12%, -20.0%, -6.7% to TFP growth, when ψ, ξ, χ



Note: The blue circle represents the calibrated parameter value of the Phillipines. In Panels A3, B3, and C3, the left y-axis refers to the interest rate spread (corresponding to the blue solid line); the right y-axis refers to the equilibrium deposit rate (corresponding to the black dashed line).

Figure 3: The impact of financial liberalization on credit access ratio, collateral-to-loan ratio, and interest rate spread in steady states.

change from the left to the right on the x-axis, respectively. The negative contribution from the savings regime is due to a general equilibrium effect. After financial deepening has occurred, capital is more efficiently allocated towards more productive entrepreneurs,



Note: The blue circle represents the calibrated parameter value of the Phillipines. In Panels A1, B1, and C1, we plot the percent change in GDP using the leftmost point as the benchmark (i.e. $\psi = 2$, $\xi = 0.7$, $\chi = 0.6$). In Panels A2, B2, and C2, we plot the percent change in TFP using the leftmost point as the benchmark. In Panels A3, B3, and C3, the left y-axis refers to the value of income Gini (corresponding to the blue solid line); the right y-axis refers to the value of wage (corresponding to the black dashed line).

Figure 4: The impact of financial liberalization on GDP, TFP, and income inequality in steady states.

resulting in higher demand for capital and labor. The shift in demand for production inputs drives up the equilibrium interest rate (black dashed lines in Panels A3-C3 of Figure 3) and wages (black dashed lines in Panels A3-C3 of Figure 4), reducing the output of entrepreneurs in the savings regime. Moreover, as more productive entrepreneurs move to the credit regime, the average productivity of entrepreneurs who are left in the savings regime falls, contributing negatively to TFP.

The contribution to GDP and TFP growth is positive from both the intensive and extensive margins, as represented by the light grey and dark grey regions in Panels A1-C2 of Figure 4. However, we find that relaxing different financial constraints has differential implications on GDP and TFP through the two margins.

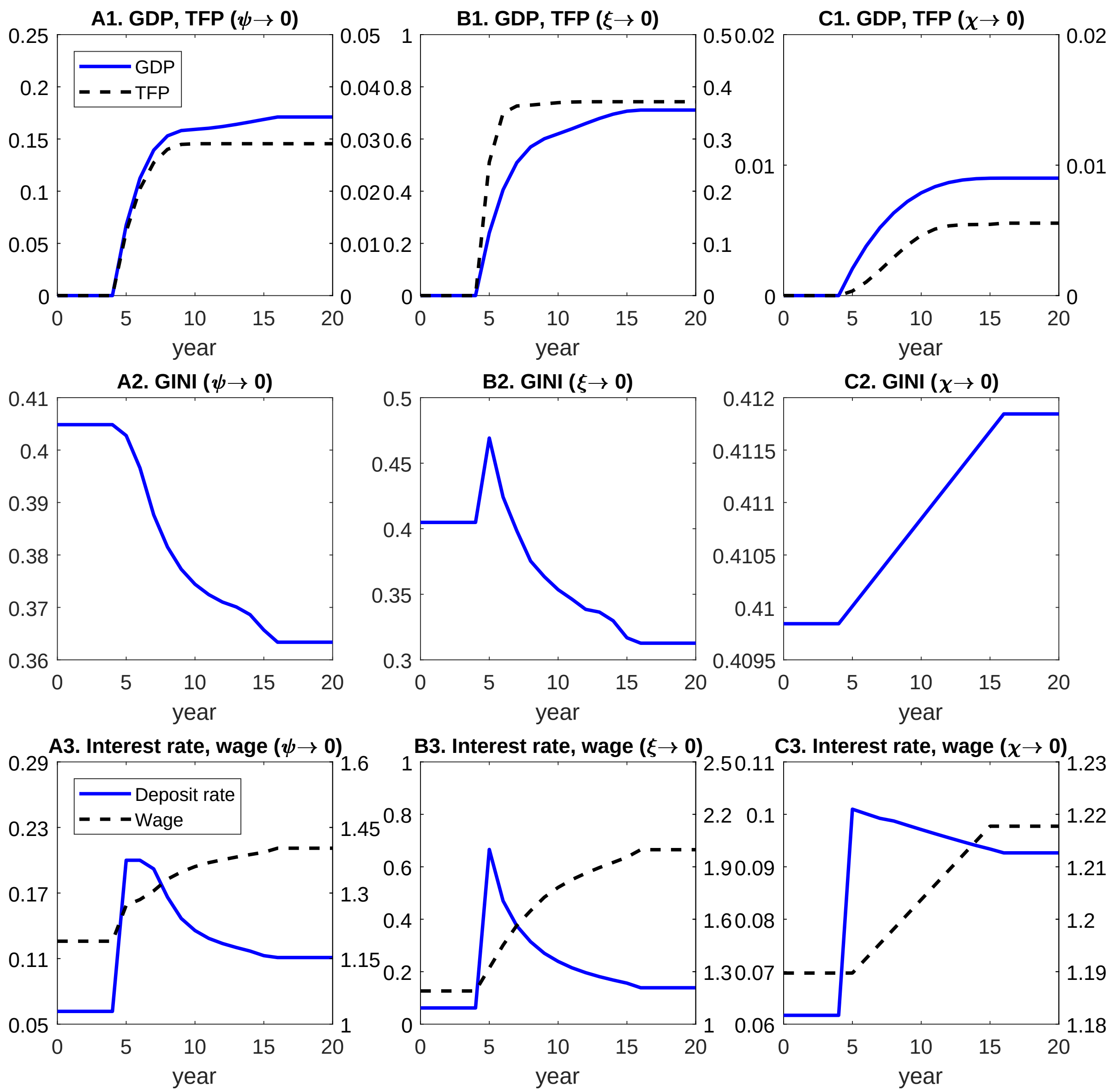
Specifically, Panels A1 and A2 of Figure 4 show that when ψ decreases from 2 to 1, GDP and TFP growth initially results from the intensive margin; and when ψ is lowered from 1 to 0, the extensive margin starts to play a role. This is because when the credit entry cost ψ falls from a relatively high value, most entrepreneurs do not have sufficient wealth to pay the upfront cost and are still excluded from the credit market. As shown in Panel A1 of Figure 3, the credit access ratio remains low when ψ is greater than 1, leading to a negligible effect on the extensive margin. However, a lower ψ increases pledgeable collateral and investment, leading to higher average output for entrepreneurs who are already in the credit regime.

By contrast, Panels B1 and B2 show that when the collateral constraint parameter ξ is lowered, both intensive and extensive margins contribute significantly to GDP and TFP growth. This is because relaxing the collateral constraint allows entrepreneurs to employ more capital in the credit regime, motivating them to pay the credit entry cost in the first place. Panels C1 and C2 show that most of the growth in GDP and TFP can be attributed to the extensive margin (but not the intensive margin), when the intermediation cost χ falls. The reason is that when the collateral constraint is unchanged, a lower intermediation cost does not allow entrepreneurs in the credit regime to expand their scale of production by borrowing more. Thus, a lower intermediation cost increases GDP through the intensive margin only by reducing the cost of production. However, the cost of production is not lowered much due to the rise in the equilibrium interest rate (the black-dashed line in Panel C3 of Figure 3), owing to more productive, but wealth constrained, entrepreneurs entering the credit regime. As a result, GDP and TFP increase mainly through the extensive margin.

In terms of inequality, Panel A3 of Figure 4 indicates that income inequality first increases and then declines as the credit entry cost ψ falls from 2 to 1. This is because when ψ decreases from a high value, only entrepreneurs already in the credit regime benefit due to lower contracting costs. Since entrepreneurs in the credit regime already have higher income than others, further increases in their incomes lead to higher income inequality. As ψ declines further, entrepreneurs in the savings regime expand their production by moving to the credit regime. The improvement in credit access generates a significant upward pressure on workers' wage (the black dashed line in Panel A3), reducing income inequality. Panel B3 shows that income inequality also first increases and then falls as the collateral constraint ξ is reduced. This is because reducing ξ from 0.7 to 0.2 initially mainly benefits entrepreneurs rather than workers, as the equilibrium wage barely responds (the black dashed line). Among entrepreneurs, reducing ξ tends to benefit those who are more productive and wealthier. When ξ is further reduced from 0.2 to 0, wealthy entrepreneurs already operate their businesses at the unconstrained scale, and thus do not benefit from a more relaxed collateral constraint. As a result, reducing ξ mainly benefits those productive, but less wealthy entrepreneurs, as well as workers due to the significant increase in equilibrium wage, lowering income inequality. By contrast, our simulation indicates that reducing the intermediation cost χ always lead to higher income inequality.

Economic Transitions We now study the transitional dynamics after financial liberalization. We start with the steady-state of the calibrated economy of the Philippines, and consider an unexpected financial deepening initiated in year 5. In Figure 5, we plot the transitional dynamics for the policy that separately reduces the three constraints, ψ (Panels A1, A2, A3), ξ (Panels B1, B2, B3), χ (Panels C1, C2, C3), to zero.

The dynamics of GDP, TFP, and Gini coefficient converge to their new steady-state values in less than 10 years. The transition period of GDP is longer than TFP, with the number of years being roughly consistent with the simulation result of Buera and Shin (2013). Interestingly, our simulation implies a significant increase in income inequality in the initial year of relaxing the collateral constraint (see year 5 in Panel B2). This is because the equilibrium interest rate is driven up dramatically by entrepreneurs' higher capital demand (see year 5 in Panel B3). The high interest rate mainly benefits wealthier entrepreneurs who already earn higher income than others, increasing income inequality. Motivated by the high interest rate, agents tend to save more. Thus, the supply of capital increases over time, driving down the interest rate and income inequality. When the credit entry cost and the intermediation cost are reduced, we also observe an overshooting in interest rates (see Panels A3 and C3). However, the magnitude of overshooting is not large enough to generate a spike in income inequality (see Panels A2 and C2).

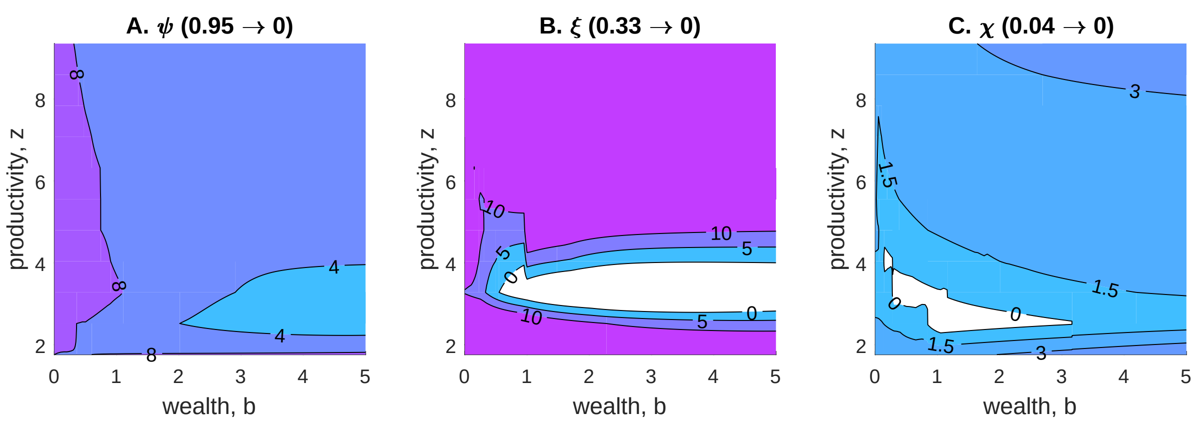


Note: In Panels A1, B1, and C1, we plot the percent change in GDP and TFP using the economy before financial liberalization as the benchmark (i.e. $\psi = 95$, $\xi = 0.33$, $\chi = 0.04$). The left y-axis refers to GDP (corresponding to the blue solid line) and the right y-axis refers to TFP (corresponding to the black dashed line). In Panels A3, B3, and C3, the left y-axis refers to the equilibrium deposit rate (corresponding to the blue solid line); the right y-axis refers to the equilibrium wage (corresponding to the black dashed line).

Figure 5: The impact of financial liberalization on GDP, TFP, and income inequality during transitions.

4.3 Distributional Implications of Financial Inclusion

In this section, we move from the macro to micro in order to shed light on the distributional welfare implications of financial inclusion policies. We conduct three counterfactual experiments for the Phillipines. In the first experiment, we assume that there is frictionless credit entry by reducing the credit entry cost ψ from the calibrated value 0.95 to 0. In the second experiment, we assume that there is frictionless borrowing by reducing the collateral constraint parameter ξ from 0.33 to 0. In the third experiment, we assume perfect financial intermediation by reducing the intermediation cost χ from 0.04 to 0. Figure 6 presents the change in steady-state welfare (consumption equivalent measure) for agents with different wealth and productivity.[8]



Note: The numbers in Panels A, B, C represent the percent change in consumption equivalent welfare in the steady state after financial deepening relative to the steady state before financial deepening. The white area represents agents associated with welfare decreases after financial deepening.

Figure 6: The steady-state welfare implications of various financial inclusion policies.

Panel A shows that reducing the credit entry cost ψ increases the welfare of all agents, but by different amounts. Agents in the region enclosed by "4" experience a relatively small welfare increase (less than 4%). These agents are entrepreneurs who are not productive but are wealthy, operating businesses at small scales and do not demand much external credit. Thus reducing the credit entry cost does not benefit these entrepreneurs much as most are currently unconstrained. A lower credit entry cost only benefits them in the future, when they get higher draws of productivity and become constrained. Agents in the region enclosed by "8" experience the largest welfare increase (more than 8%).

[8]Note that due to the existence of idiosyncratic productivity shocks, agents who are currently productive can be unproductive in the future, or vice versa. Our value functions derived from forward-looking agents already captures the effect from such dynamic changes in individual characteristics.

These agents are currently constrained workers, who are productive but poor. A lower credit entry cost allows them to access credit and start businesses, significantly increasing their life-time utility. This is what contributes to the lower income inequality observed in Panel A3 of Figure 4. The agents in the middle region experience a modest increase in welfare, between 4% and 8%. These agents are currently either workers who experience an increase in welfare due to higher wages, or constrained entrepreneurs who expand their businesses after saving the credit entry cost.

In contrast to Panel A, we find that not everyone gains when the collateral constraint is relaxed or the intermediation cost is reduced. Panel B shows that when the collateral constraint is relaxed, agents in the (white) region enclosed by "0" incur welfare losses. These are currently entrepreneurs who are moderately productive and wealthy. Their businesses are operated at the unconstrained scale, and thus further relaxing the collateral constraint does not benefit them. However, relaxing the collateral constraint results in a general equilibrium effect that increases the cost of production (i.e. interest rate and wage), reducing their profits. Our simulation implies that the biggest winners in this experiment are very productive entrepreneurs (top region) and very unproductive workers (bottom region), whose welfare increase by more than 10%. The former group benefits from increased scale of business and the latter group benefits from doubling of the equilibrium wage.

Panel C shows that when the intermediation cost χ decreases, agents who are currently moderately productive and poor (in the region enclosed by "0") incur welfare losses. These agents are entrepreneurs who operate their businesses at small scales due to low productivity. They do not demand much credit and receive little benefit from a lower intermediation cost. However, they suffer from higher production costs due to the rise in the equilibrium interest rate and wage. The agents who receive the largest welfare gains (more than 3%) are those in the upper-right region and in the bottom-right region, enclosed by "3". The former group consists of constrained entrepreneurs who borrow the largest amount of credit. Thus, a lower intermediation cost significantly reduces their cost of production. The latter group consists of wealthy workers, who receive significantly more interest from their savings when the equilibrium interest rate goes up. In both Panels B and C, our simulation indicates that the currently most productive and wealthy entrepreneurs also receive the largest welfare gains after financial inclusion. This explains why we observe a steadily increase income inequality in Panels B3 and C3 of Figure 4.

4.4 Equilibrium Interactions Among Financial Constraints

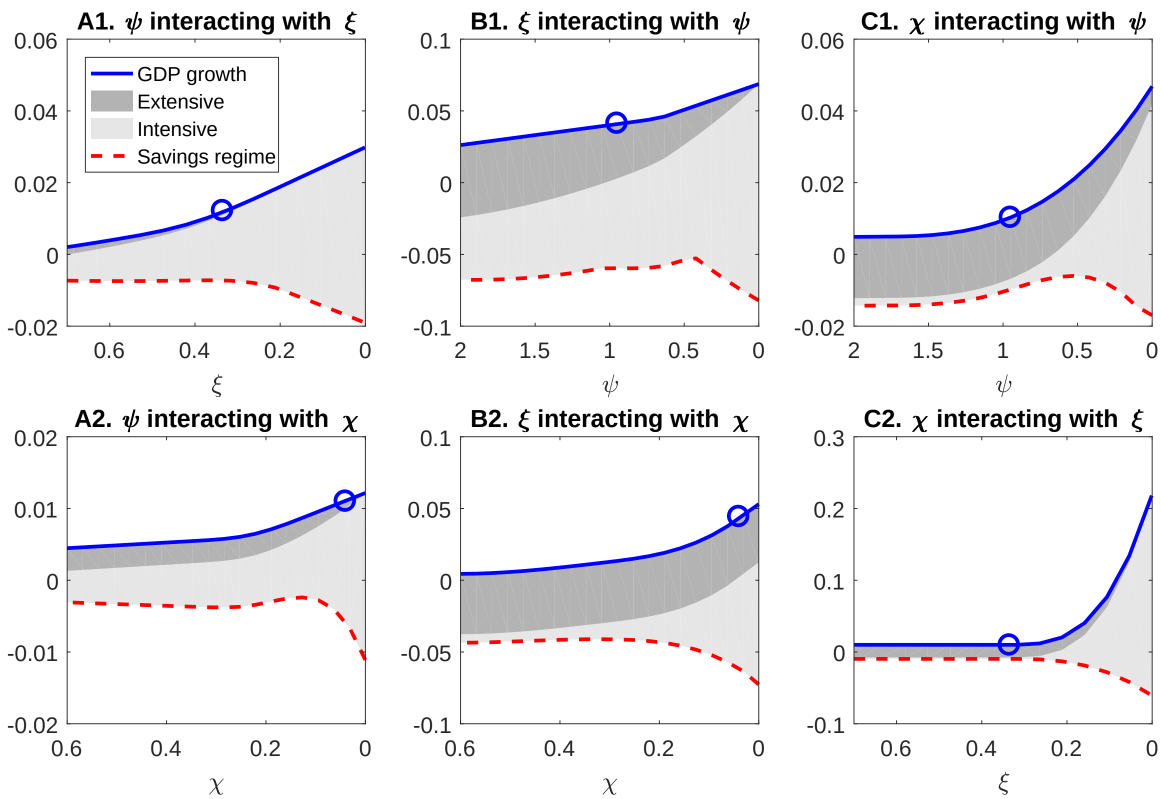
Our previous analysis indicates that financial inclusion policies targeted at alleviating different financial frictions may have differential effects on GDP, TFP, and income inequality. In principle, countries may also adopt more comprehensive financial deepening that alleviates multiple sources of financial frictions at the same time. What would be the potential effects on GDP and through which margins? To what extent does the effect of targeting one financial constraint depend on the tightness of the other financial constraints? In this section, we use our calibrated model of the Phillipines to investigate the interactions among the different financial constraints in general equilibrium.

We conduct a sequence of counterfactual experiments in which we measure the percent growth in steady-state GDP after relaxing one financial constraint (target constraint) conditional on different values of the second constraint (moving constraint), while at the same time keeping the third constraint unchanged. Effectively, our experiments measure the interaction effect between the target and the moving financial constraints. For the three financial constraints (ψ, ξ, χ), there are six combinations in total as presented in Figure 7.

In partial equilibrium, our theory suggests that alleviating two constraints jointly tends to have amplifying effects through the intensive margin and dampening effects through the extensive margin (see Propositions 2 and 3). Our simulations verify these results in general equilibrium.

In particular, Panel A1 considers ψ as the target constraint and ξ as the moving constraint. On the y-axis, we plot the increase in GDP (blue solid line) resulting from lowering the target constraint ψ from its calibrated value 0.95 to 0.8 for different values of the moving constraint ξ (represented on the x-axis). The dark-grey, light-grey areas, and the red-dashed line represent the increase in GDP attributed to the extensive margin, the intensive margin, and the savings regime. Moving from the left to the right along the x-axis, as the collateral constraint becomes more relaxed (lower ξ), we observe that the intensive-margin effect increases while the extensive-margin effect diminishes. Interestingly, we find that the interaction effect through the intensive margin dominates the effect through the extensive margin. That is, when we relax the collateral constraint, the (increasing) intensive margin effect dominates the (decreasing) extensive margin effect. As a result, the increase in GDP from lowering ψ is larger when the collateral constraint is more relaxed (see the upward sloping blue solid line).

Similarly, in Panel A2, we hold ψ as the target constraint and consider χ as the moving constraint. Again, as we move from the left to the right along the x-axis (lower χ), the intensive margin effect increases whereas the extensive margin effect diminishes. The



Note: Panels A1 and A2 present the percent growth in steady-state GDP when ψ is reduced from the calibrated value 0.95 to 0.8. Panel A1 studies its interaction effect with ξ by varying ξ from 0.7 to 0. Panel A2 studies its interaction effect with χ by varying χ from 0.6 to 0. Panels B1 and B2 present the percent growth in steady-state GDP when ξ is reduced from the calibrated value 0.33 to 0.29. Panel B1 studies its interaction effect with ψ by varying ψ from 2 to 0. Panel B2 studies its interaction effect with χ by varying χ from 0.6 to 0. Panels C1 and C2 present the percent growth in steady-state GDP when χ is reduced from the calibrated value 0.04 to 0. Panel C1 studies its interaction effect with ψ by varying ψ from 2 to 0. Panel C2 studies its interaction effect with ξ by varying ξ from 1 to 0.3.

Figure 7: The impact of financial liberalization on steady-state GDP conditional on different financial system characteristics.

former dominates the latter, resulting in a larger increase in GDP from lowering ψ. These patterns are also observed in panels B1 and B2, where we consider a relaxation of the target constraint ξ from 0.33 to 0.29, as well as in panels C1 and C2 for reducing the target constraint χ from 0.04 to 0.

Although we cannot analytically establish the result that the interaction effect through the intensive margin dominates the extensive margin, we find that the result is robust for different combinations of policy parameters and across models calibrated to match data for different countries. This implies that the net interaction effect on GDP is positive. That is, when we alleviate multiple financial constraints together, the net effect on GDP is a convex combination of the effect from relaxing each financial constraint separately. Intuitively, this is because there is a diminishing return from alleviating the same financial

Table 2: Country characteristics.

	Emerging market economies						Low-income countries					
	Pakistan		Bangladesh		Brazil		Uganda		Nepal		Kyrgyzstan	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
Firms with credit (%)	6.7	6.7	34.1	34.7	59.2	60.0	9.7	9.5	35.0	35.4	29.2	29.0
Collateral (% of loans)	153.4	153.4	271.1	271.4	95.1	100.0	161.8	160.9	364.2	363.9	187.7	185.5
Interest rate spread (%)	4.8	4.7	1.9	1.9	35.4	33.8	10.9	10.3	4.4	4.2	21.3	20.8
Real interest rate (%)	0.2	0.3	4.5	4.5	2.9	2.5	8.1	7.9	-3.0	-2.9	-1.0	-1.1
Income GINI coefficient	0.31	0.30	0.32	0.33	0.54	0.49	0.41	0.40	0.33	0.31	0.29	0.30
Top 5% employment share (%)	65.7	63.2	55.4	51.2	63.1	68.4	63.6	61.8	47.6	46.5	48.3	44.5
Top 10% employment share (%)	77.6	76.8	71.2	69.5	74.6	77.3	73.8	74.2	65.1	62.1	58.7	55.9
Top 20% employment share (%)	87.6	88.1	86.2	82.3	84.7	86.2	82.9	81.5	79.0	77.2	72.0	69.7
Top 40% employment share (%)	94.7	91.5	95.2	96.7	93.1	97.1	91.2	92.3	89.9	90.1	85.7	87.8
Gross savings (% of GDP)	21.4	20.4	21.2	20.6	16.9	15.9	20.8	21.5	43.8	40.8	15.6	15.2

friction. To elaborate, consider a country with both a very tight collateral constraint and a very high credit entry cost. Relaxing the collateral constraint will initially have a significant effect on boosting the country's GDP as this is the most binding financial friction. However, when the collateral constraint is not binding, further relaxing it has little effect on GDP. In this case, reducing the credit entry cost will have a greater effect on GDP as the credit entry cost now becomes the bottleneck constraining financial inclusion. On the other hand, if we start with reducing the credit entry cost, the initial effect on GDP would also diminish, as the collateral constraint becomes the bottleneck.

4.5 Identifying the Most Binding Constraint

The interaction effects among the three financial constraints suggest that effective financial inclusion policies should be designed to target the most binding constraint. Moreover, policies that are effective in generating GDP growth in one country may not be as equally effective in other countries. Our model provides a framework to systematically study the potential effects from different financial inclusion policies, and hence, to identify the bottleneck constraint in a financial system. As an illustration, we now consider six representative countries and use the model to evaluate the effectiveness of conducting financial inclusion policies in each country.

We calibrate our model to match the statistics of three emerging market economies (Pakistan, Bangladesh, Brazil), and three low-income countries (Uganda, Nepal, and Kyrgyzstan). As shown in Table 2, these countries' financial systems exhibit different characteristics. For example, fewer than 6.7% of firms in Pakistan have access to bank credit as compared to 34.1% and 59.2% in Bangladesh and Brazil, respectively. However, among those firms with credit access, firms in Bangladesh have to post collateral that amounts to 271.1% of the face value of loans, as compared to 153.4% and 95.1% in Pakistan and Brazil, respectively. Surprisingly, although firms in Brazil tend to have access to bank credit and face a less tight collateral constraint, the loan interest rate is higher than the deposit rate by 35.4 percentage points! Such a sharp distinction is also observed among the low-income countries. Uganda has the lowest credit access ratio but has relatively more relaxed collateral constraint. Nepal has the lowest interest rate spread but requires substantially more collateral for borrowing. Kyrgyzstan has relatively high credit access and a lower collateral requirement, but charges a very high interest rate spread.

Table 3: The impact of financial inclusion of various forms on steady-state GDP, TFP and income inequality.

	$\psi \to 0$			$\xi \to 0$			$\chi \to 0$		
	GDP(%)	TFP(%)	Gini	GDP(%)	TFP(%)	Gini	GDP(%)	TFP(%)	Gini
Pakistan	18.2	7.0	-0.041	12.7	10.5	-0.011	0.5	0.3	0.003
Bangladesh	7.9	3.1	-0.017	62.1	51.3	-0.050	0.2	0.1	0.004
Brazil	3.5	1.4	-0.009	0	0	0.000	15.4	8.4	0.018
Uganda	17.5	6.8	-0.033	16.7	13.5	-0.023	2.3	0.9	0.006
Nepal	10.8	4.2	-0.025	71.4	57.2	-0.048	0.6	0.4	0.003
Kyrgyzstan	6.6	1.7	-0.012	10.5	7.7	-0.009	13.9	5.6	0.027

The comparison in Table 2 suggests that financial deepening is far from uniform along the various dimensions. Despite the existence of such heterogeneity, our model is able to match a country's financial system, as reflected by the three moments. We now use the model to evaluate the potential effect of various financial inclusion policies. Table 3 presents the changes in steady-state GDP, TFP and the Gini coefficient when the six countries adopt the best-possible financial intermediation technology on a single dimension (i.e. $\psi = 0$, $\xi = 0$, or $\chi = 0$).

The model implies that Pakistan's GDP and TFP would increase by 18.2% and 7.0% when the credit entry cost ψ goes to 0, while keeping ξ and χ unchanged. Relaxing the collateral constraint ξ to 0 boosts GDP and TFP by 12.7% and 10.5%, while reducing the intermediation cost χ to 0 increases GDP and TFP by only 0.5% and 0.3%. Therefore, our model suggests that the most binding financial constraint for Pakistan arises from high credit entry cost, which is also reflected by the low credit access ratio as only 6.7% of firms have bank credit.

By contrast, the model implies that Bangladesh's most binding financial friction is the tight collateral constraint. If we relax ξ to 0 in Bangladesh, GDP and TFP would increase by 62.1% and 51.3%, while reducing either ψ or χ to 0 only increases GDP by 7.9% and 0.2%, respectively. Regarding Brazil, our model implies that the high intermediation cost is the key friction that hinders economic growth. Reducing χ to 0 increases Brazil's GDP and TFP by 15.4% and 8.4%, while reducing ψ to 0 only increases GDP and TFP by 3.5%

and 1.4%.

A similar comparison is also observed among the three low-income countries. Specifically, our model implies that the high credit entry cost is the key bottleneck in Uganda, while tight collateral constraint and high intermediation cost hinder financial inclusion in Nepal and Kyrgyzstan.

Perhaps it is not surprising that we obtain these results given that these countries differ in terms of the extent of financial development. A simple eyeball test of the credit access ratio, collateral-to-loan ratio, and interest rate spread from Table 2 would allow policy makers to recognize which financial constraint is most binding. However, it does not invalidate the model's ability to provide useful policy guidance. In fact, the six countries we cherry picked represent those with extremely unbalanced financial systems. In reality, as most countries do not share these extreme characteristics, it could be difficult to figure out the key bottleneck. Our model provides a framework to systematically think of these questions and to quantify the implication of financial inclusion policies that alleviate different financial frictions.

5 Conclusion

In this paper, we develop a tractable general equilibrium model with heterogeneous agents to study how different sources of financial frictions interact in equilibrium. Our model highlights three different dimensions of a financial system: breath, depth, and efficiency. These characteristics reflect different sources of financial constraints whose tightness can be captured by three indicators, credit access ratio, loan-to-collateral ratio, and interest rate spreads.

Our headline results indicate that the three financial constraints interact with each other and have important policy implications. Analytically, we show that relaxing these constraints increases GDP through both the extensive margin and intensive margin. However, the interaction tends to dampen the effect through the extensive margin while amplify the effect through the intensive margin. Quantitatively, we find that the interaction effect on the intensive margin tends to dominate, indicating that more effective financial inclusion policies should be designed to balance the development of different dimensions of a financial system. In particular, by conducting comparative statics, our calibrated model is able to provide suggestive guidelines on financial inclusion policies by identifying the most binding constraint.

References

Abraham, Arpad, and Nicola Pavoni. 2005. "The Efficient Allocation of Consumption under Moral Hazard and Hidden Access to the Credit Market." *Journal of the European Economic Association*, 3(2-3): 370–381.

Aghion, Philippe, and Patrick Bolton. 1997. "A Theory of Trickle-Down Growth and Development." *Review of Economic Studies*, 64(2): 151–72.

Aguirregabiria, Victor, Robert Clark, and Hui Wang. 2017. "The Geographic Flow of Bank Funding and Access to Credit: Branch Networks and Local-Market Competition."

Albuquerque, Rui, and Hugo A. Hopenhayn. 2004. "Optimal Lending Contracts and Firm Dynamics." *Review of Economic Studies*, 71(2): 285–315.

Alem, Mauro, and Robert M. Townsend. 2013. "An Evaluation of Financial Institutions: Impact on Consumption and Investment Using Panel Data and the Theory of Risk-Bearing." *Annals Issue of the Journal of Econometrics in Honor of Bill Barnett*, , (forthcoming).

Amaral, Pedro S., and Erwan Quintin. 2010. "Limited Enforcement, Financial Intermediation, And Economic Development: A Quantitative Assessment." *International Economic Review*, 51(3): 785–811.

Assuncao, Juliano, Sergey Mityakov, and Robert M. Townsend. 2012. "Ownership Matters: the Geographical Dynamics of BAAC and CommercialBanks in Thailand." Working Paper.

Banerjee, Abhijit V, and Andrew F Newman. 1993. "Occupational Choice and the Process of Development." *Journal of Political Economy*, 101(2): 274–98.

Banerjee, Abhijit V., and Benjamin Moll. 2010. "Why Does Misallocation Persist?" *American Economic Journal: Macroeconomics*, 2(1): 189–206.

Buera, Francisco J., and Yongseok Shin. 2011. "Self-insurance vs. self-financing: A welfare analysis of the persistence of shocks." *Journal of Economic Theory*, 146(3): 845–862.

Buera, Francisco J., and Yongseok Shin. 2013. "Financial Frictions and the Persistence of History: A Quantitative Exploration." *Journal of Political Economy*, 121(2): 221 – 272.

Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin. 2011. "Finance and Development: A Tale of Two Sectors." *American Economic Review*, 101(5): 1964–2002.

Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin. 2012. "The Macroeconomics of Microfinance." National Bureau of Economic Research, Inc NBER Working Papers 17905.

Burgess, Robin, and Rohini Pande. 2005. "Do Rural Banks Matter? Evidence from the Indian Social Banking Experiment." *American Economic Review*, 95(3): 780–795.

Cagetti, Marco, and Mariacristina De Nardi. 2006. "Entrepreneurship, Frictions, and Wealth." *Journal of Political Economy*, 114(5): 835–870.

Caselli, Francesco, and Nicola Gennaioli. 2013. "Dynastic Management." *Economic Inquiry*, 51(1): 971–996.

Castaneda, Ana, Javier Diaz-Gimenez, and Jose-Victor Rios-Rull. 2003. "Accounting for the U.S. Earnings and Wealth Inequality." *Journal of Political Economy*, 111(4): 818–857.

Celerier, Claire, and Adrien Matray. 2017. "Bank Branch Supply and the Unbanked Phenomenon."

Clementi, Gina Luca, and Hugo A Hopenhayn. 2006. "A Theory of Financing Constraints and Firm Dynamics." *The Quarterly Journal of Economics*, 121(1): 229–265.

Dabla-Norris, Era, Yan Ji, Robert M. Townsend, and D. Filiz Unsal. 2014. "Identifying Financial Constraints on Growth, and Inequality: A Structural Framework for Policy Recommendation in Developing Countries."

Dabla-Norris, Era, Yan Ji, Robert M. Townsend, and D. Filiz Unsal. 2015. "Distinguishing Constraints on Financial Inclusion and Their Impact on GDP, TFP, and Inequality." NBER Working Paper No. 20821,.

Davies, James B. 1982. "The Relative Impact of Inheritance and Other Factors on Economic Inequality." *The Quarterly Journal of Economics*, 97(3): 471–98.

Doepke, Matthias, and Robert M. Townsend. 2006. "Dynamic Mechanism Design with Hidden Income and Hidden Actions." *Journal of Economic Theory*, 126(1): 235–285.

Gilje, Erik P., Elena Loutskina, and Philip E. Strahan. 2016. "Exporting Liquidity: Branch Banking and Financial Integration." *Journal of Finance*, 71(3): 1159–1184.

Gine, Xavier, and Robert M. Townsend. 2004. "Evaluation of financial liberalization: a general equilibrium model with constrained occupation choice." *Journal of Development Economics*, 74(2): 269–307.

Greenwood, Jeremy, and Boyan Jovanovic. 1990. "Financial Development, Growth, and the Distribution of Income." *Journal of Political Economy*, 98(5): 1076–1107.

Greenwood, Jeremy, Juan M. Sanchez, and Cheng Wang. 2010. "Financing Development: The Role of Information Costs." *American Economic Review*, 100(4): 1875–91.

Greenwood, Jeremy, Juan Sanchez, and Cheng Wang. 2013. "Quantifying the Impact of Financial Development on Economic Development." *Review of Economic Dynamics*, 16(1): 194–215.

Hsieh, Chang-Tai, and Peter J. Klenow. 2009. "Misallocation and Manufacturing TFP in China and India." *The Quarterly Journal of Economics*, 124(4): 1403–1448.

Huggett, Mark. 1996. "Wealth Distribution in Life-cycle Economies." *Journal of Monetary Economics*, 38(3): 469–494.

Jeong, Hyeok, and Robert M. Townsend. 2008. "Growth And Inequality: Model Evaluation Based On An Estimation-Calibration Strategy." *Macroeconomic Dynamics*, 12(S2): 231–284.

Jeong, Hyeok, and Robert Townsend. 2007. "Sources of TFP growth: occupational choice and financial deepening." *Economic Theory*, 32(1): 179–221.

Jermann, Urban, and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review*, 102(1): 238–271.

Karaivanov, Alexander, and Robert M. Townsend. 2014. "Dynamic Financial Constraints: Distinguishing Mechanism Design From Exogenously Incomplete Regimes." *Econometrica*, 82(3): 887–959.

Kinnan, Cynthia. 2014. "Distinguishing Barriers to Insurance in Thai Villages." Northwestern University Working Papers 1276.

Lloyd-Ellis, Huw, and Dan Bernhardt. 2000. "Enterprise, Inequality and Economic Development." *Review of Economic Studies*, 67(1): 147–68.

Martin, Alberto, and Filippo Taddei. 2013. "International Capital Flows and Credit Market Imperfections: A Tale of Two Frictions." *Journal of International Economics*, 89(2): 441–452.

Midrigan, Virgiliu, and Daniel Yi Xu. 2014. "Finance and Misallocation: Evidence from Plant-Level Data." *American Economic Review*, 104(2): 422–58.

Moll, Benjamin. 2014. "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?" *American Economic Review*, , (forthcoming).

Moll, Benjamin, Robert M. Townsend, and Victor Zhorin. 2014. "Economic Development and the Equilibrium Interaction of Financial Frictions." Working Papers.

Nardi, Mariacristina De. 2004. "Wealth Inequality and Intergenerational Links." *Review of Economic Studies*, 71(3): 743–768.

Nguyen, Hoai-Luu Q. 2017. "Are Credit Markets Still Local? Evidence from Bank Branch Closings."

Okuda, Hidenobu, and Fumiharu Mieno. 1999. "What Happened to Thai Commercial Banks in the Pre-Asian Crisis Period: Microeconomic Analysis of Thai Banking Industry." *Hitotsubashi Journal of Economics*, 40(2): 97–121.

Paulson, Anna L., Robert M. Townsend, and Alexander Karaivanov. 2006. "Distinguishing Limited Liability from Moral Hazard in a Model of Entrepreneurship." *Journal of Political Economy*, 114(1): 100–144.

Townsend, Robert. 1983*a*. "Financial Structure and Economic Activity." *American Economic Review*, 73: 895–911.

Townsend, Robert. 1983*b*. "Theories of Intermediated Structures." *Carnegie-Rochester Conf. Ser. Public Policy*, 18: 221–272.

Townsend, Robert M. 1979. "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory*, 21(2): 265–293.

Townsend, Robert M., and Kenichi Ueda. 2006. "Financial Deepening, Inequality, and Growth: A Model-Based Quantitative Evaluation." *Review of Economic Studies*, 73(1): 251–293.

Online Appendix (Not for Publication)

A GDP Formula

We can rewrite equation (2.15) as the sum of workers' and entrepreneurs' income

$$\begin{aligned}\text{GDP}_\Omega &= \iint_{(b,z)\in\Phi} I(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \iint_{(b,z)\in\Phi^E} \delta k(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z \\ &= \iint_{(b,z)\notin\Phi^E} I(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \iint_{(b,z)\in\Phi^E} [I(b,z;\Omega)+\delta k(b,z;\Omega)]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\end{aligned} \tag{A.1}$$

Substituting equation (2.14) into equation (A.1), we obtain

$$\begin{aligned}\text{GDP}_\Omega = &\iint_{(b,z)\notin\Phi^E} (w+rb)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z \\ &+ \iint_{(b,z)\in\Phi^S} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha})^{1-\nu} - wl(b,z;\Omega) + r(b-k(b,z;\Omega))]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z \\ &+ \iint_{(b,z)\in\Phi^C} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha})^{1-\nu} - wl(b,z;\Omega) - \psi - (r+\chi)(k(b,z;\Omega)-b+\psi)]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\end{aligned} \tag{A.2}$$

Taking out the terms with wage w and interest rate r, equation (A.2) is rewritten as

$$\begin{aligned}\text{GDP}_\Omega =& w\left[\iint_{(b,z)\notin\Phi^E} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z - \iint_{(b,z)\in\Phi^E} l(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\right] \\ &+ r\left[\iint_{(b,z)\notin\Phi^E} bh(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \iint_{(b,z)\in\Phi^S} [b-k(b,z;\Omega)]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\right. \\ &\left. - \iint_{(b,z)\in\Phi^C} [k(b,z;\Omega)-b+\psi]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\right] \\ &+ \iint_{(b,z)\in\Phi^S} z[k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha}]^{1-\nu}h(b,z;\Omega)\mathrm{d}b\mathrm{d}z \\ &+ \iint_{(b,z)\in\Phi^C} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha})^{1-\nu} - \psi - \chi(k(b,z;\Omega)-b+\psi)]h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\end{aligned} \tag{A.3}$$

The labor and capital market clearing conditions (2.10-2.11) imply that the first two terms on the right-hand side of equation (A.3) are zero, thus we have

$$
\begin{aligned}
\mathrm{GDP}_\Omega = & \iint\limits_{(b,z)\in\Phi^S} z[k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha}]^{1-\nu} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z \\
& + \iint\limits_{(b,z)\in\Phi^C} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha})^{1-\nu} - \psi - \chi(k(b,z;\Omega) - b + \psi)] h(b,z;\Omega)\mathrm{d}b\mathrm{d}z \\
= & \iint\limits_{(b,z)\in\Phi^S} y^S(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \iint\limits_{(b,z)\in\Phi^C} y^C(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z.
\end{aligned}
\tag{A.4}
$$

B Proofs

B.1 Proof of Proposition 1

Consider the entrepreneur's problem, labor is a freely adjustable input. Taking the FOC with respect to l_t, we can derive the optimal labor demand as a function of capital k_t and productivity z_t

$$
l_t(z_t,k_t) = \left[\frac{(1-\alpha)(1-\nu)z_t k_t^{\alpha(1-\nu)}}{w_t}\right]^{\frac{1}{\nu+\alpha(1-\nu)}} \tag{B.1}
$$

Because there is no uncertainty within each period, maximizing utility is equivalent to maximizing the end-of-period wealth $\tilde{b}_t$, defined by $\tilde{b}_t = c_t + b_{t+1}$. Substituting optimal labor demand (B.1) into the entrepreneur's budget constraint, we obtain

$$
\begin{aligned}
\tilde{b}_t(b_t,z_t,k_t) &= z_t(k_t^\alpha l_t^{1-\alpha})^{1-\nu} + (1-\delta)k_t - w_t l_t + (1+r_t)(b_t - k_t) \\
&= \left[\frac{(1-\alpha)(1-\nu)z_t}{w_t}\right]^{\frac{1}{\nu+\alpha(1-\nu)}} k_t^{\frac{\alpha(1-\nu)}{\nu+\alpha(1-\nu)}} \frac{\nu+\alpha(1-\nu)}{(1-\nu)(1-\alpha)} w_t + (1-\delta)k_t + (1+r_t)(b_t-k_t).
\end{aligned}
\tag{B.2}
$$

In the absence of financial frictions, capital k_t is optimally chosen to maximize $\tilde{b}_t$. By taking the FOC with respect to k_t, we derive the optimal (unconstrained) capital demand k_t^u:

$$
k_t^u(z_t) = \left[\frac{\alpha w_t}{(1-\alpha)(r_t+\delta)}\right]^{\frac{\nu+\alpha(1-\nu)}{\nu}} \left[\frac{(1-\alpha)(1-\nu)z_t}{w_t}\right]^{\frac{1}{\nu}} \tag{B.3}
$$

Substituting equation (B.3) into equation (B.2), we obtain the maximal end-of-period wealth $\tilde{b}^e_t$ for being an entrepreneur:

$$\tilde{b}^e_t(b_t, z_t) = (1+r_t)b_t + z_t^{\frac{1}{\nu}} [1-\alpha(1-\nu)] \left[\frac{\alpha w_t}{(1-\alpha)(r_t+\delta)}\right]^{\frac{\alpha(1-\nu)}{\nu}} \left[\frac{(1-\alpha)(1-\nu)}{w_t}\right]^{\frac{1-\nu}{\nu}} \tag{B.4}$$

The end-of-period wealth for being a worker is $\tilde{b}^w_t(b_t) = (1+r_t)b_t + w_t$. Since $\tilde{b}^e_t(b_t, z_t)$ increases with z_t, the threshold of of entrepreneurial productivity $\underline{z}_t$ at time t is given by $\tilde{b}^w_t(b_t) = \tilde{b}^e_t(b_t, \underline{z}_t)$,

$$\underline{z}_t = \frac{(r_t+\delta)^{\alpha(1-\nu)} w_t^{1-\alpha(1-\nu)}}{[1-\alpha(1-\nu)]^{\nu}[\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1-\nu)]^{1-\nu}} \tag{B.5}$$

B.2 Proof of Proposition 2

With $\nu = 0$, production technology has constant return to scale and the borrowing constraint is binding, i.e. $k^s_t = b_t$ for an entrepreneur in the savings regime and $k^c_t = \frac{b_t - \psi}{\xi}$ for an entrepreneur in the credit regime. Given z_t, b_t, the entrepreneur's utility is increasing in current consumption c_t and the future value $V_{t+1}(z_{t+1}, b_{t+1})$, which increases with b_{t+1} for every realization of z_{t+1}. Therefore, the optimal decision to borrow is determined to maximize the end-of-period wealth $\tilde{b}^e_t = c_t + b_{t+1}$.

Now let us focus on the steady state. Consider an entrepreneur of (b, z), since labor supply is not subject to the collateral constraint, the optimal labor supply l is linear in k

$$l(z,k) = \left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} k. \tag{B.6}$$

Substituting $l(z,k)$, we derive output as a linear function of k,

$$f(k,z) = zk^{\alpha}l^{1-\alpha} = z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} k \tag{B.7}$$

With constant return to scale, optimal capital demand has a bang-bang solution. If the agent chooses to be an entrepreneur, then $k = b$ in the savings regime and $k = \frac{b-\psi}{\xi}$ in the credit regime. Substituting equation (B.7) into the budget constraint, we derive the end-of-period wealth in the savings regime as

$$\begin{aligned}\tilde{b}^{e,s}(b,z) &= zk^{\alpha}l^{1-\alpha} + (1-\delta)k - wl + (1+r)(b-k) \\ &= z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} b + (1-\delta)b - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} b,\end{aligned} \tag{B.8}$$

and in the credit regime as

$$
\begin{aligned}
\tilde{b}^{e,c}(b,z) =& z(k^{\alpha} l^{1-\alpha})^{1-\nu} + (1-\delta)k - wl - (1+r+\chi)(k-b+\psi) \\
=& z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} k + (1-\delta)k - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} k - (1+r+\chi)(k-b+\psi) \\
=& z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} \frac{b-\psi}{\xi} + \frac{(1-\delta)(b-\psi)}{\xi} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} \frac{b-\psi}{\xi} \\
& - (1+r+\chi)\left(\frac{b-\psi}{1-\xi} - b + \psi\right)
\end{aligned} \tag{B.9}
$$

The threshold of wealth $\underline{b}(z;\Omega)$ is determined by $\tilde{b}^{e,s}(z,\underline{b}) = \tilde{b}^{e,c}(z,\underline{b})$:

$$
\underline{b}(z;\Omega) = \frac{\psi}{1-\xi} + \frac{\psi}{1-\xi} \frac{(1+r+\chi)\xi}{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi}. \tag{B.10}
$$

Taking the first derivative, we derive the effect through the extensive margin from relaxing each financial constraint separately:

$$
\frac{\partial \bar{b}}{\partial \psi} = \frac{1}{1-\xi} + \frac{1}{1-\xi} \frac{(1+r+\chi)\xi}{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi} \geq 0; \tag{B.11}
$$

$$
\frac{\partial \bar{b}}{\partial \xi} = \frac{\psi}{(1-\xi)^2} + \frac{\psi}{(1-\xi)^2} \frac{1+r+\chi}{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi} \geq 0; \tag{B.12}
$$

$$
\frac{\partial \bar{b}}{\partial \chi} = \frac{\psi\xi}{1-\xi} \frac{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} + 1 - \delta}{\left[z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi\right]^2} \geq 0. \tag{B.13}
$$

Taking the cross-partial derivative, we derive the interaction effect through the extensive margin from relaxing two financial constraints jointly:

$$
\frac{\partial^2 \bar{b}}{\partial \psi \partial \xi} = \frac{\partial^2 \bar{b}}{\partial \xi \partial \psi} = \frac{1}{(1-\xi)^2} + \frac{1}{(1-\xi)^2} \frac{1+r_t+\chi}{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi} \geq 0; \tag{B.14}
$$

$$\frac{\partial^2 \overline{b}}{\partial \xi \partial \chi} = \frac{\partial^2 \overline{b}}{\partial \chi \partial \xi} = \frac{\psi}{(1-\xi)^2} \frac{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w} + 1 - \delta\right]^{\frac{1}{\alpha}}}{\left[z_t\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi\right]^2} \geq 0; \quad \text{(B.15)}$$

$$\frac{\partial^2 \overline{b}}{\partial \chi \partial \psi} = \frac{\partial^2 \overline{b}}{\partial \psi \partial \chi} = \frac{\xi}{1-\xi} \frac{z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} + 1 - \delta}{\left[z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - w\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1}{\alpha}} - \delta - r - \chi\right]^2} \geq 0. \quad \text{(B.16)}$$

B.3 Proof of Proposition 3

Substituting optimal labor demand (B.6) and $k^c = \frac{b-\psi}{\xi}$ into equation (2.18), we have

$$y^C(b, z; \Omega) = \left[z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - \chi\right] \frac{b-\psi}{\xi} + \chi(b-\psi) - \psi. \quad \text{(B.17)}$$

Taking the first derivative, we derive the effect through the intensive margin from relaxing each financial constraint separately:

$$\frac{\partial y^C}{\partial \psi} = -\left[z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - \chi\right] \frac{1}{\xi} + \chi(b-\psi) - 1 \leq 0; \quad \text{(B.18)}$$

$$\frac{\partial y^C}{\partial \xi} = -\left[z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - \chi\right] \frac{b-\psi}{\xi^2} \leq 0; \quad \text{(B.19)}$$

$$\frac{\partial y^C}{\partial \chi} = -\frac{(1-\xi)(b-\psi)}{\xi} \leq 0. \quad \text{(B.20)}$$

Taking the cross-partial derivative, we derive the interaction effect through the intensive margin from relaxing two financial constraints jointly:

$$\frac{\partial^2 y^C}{\partial \psi \partial \xi} = \frac{\partial^2 y^C}{\partial \xi \partial \psi} = \left[z\left[\frac{(1-\alpha)z}{w}\right]^{\frac{1-\alpha}{\alpha}} - \delta - \chi\right] \frac{1}{\xi^2} \geq 0; \quad \text{(B.21)}$$

$$\frac{\partial^2 y^C}{\partial \xi \partial \chi} = \frac{\partial^2 y^C}{\partial \chi \partial \xi} = \frac{b-\psi}{\xi^2} \geq 0; \quad \text{(B.22)}$$

$$\frac{\partial^2 y^C}{\partial \chi \partial \psi} = \frac{\partial^2 y^C}{\partial \psi \partial \chi} = \frac{1-\xi}{\xi} \geq 0. \quad \text{(B.23)}$$

B.4 Proof of Proposition 4

According to equation (2.16) and the definition of $\underline{b}(z;\Omega)$, GDP can be written as

$$\mathrm{GDP}_\Omega = \int_z \int_0^{\underline{b}(z;\Omega)} y^S(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z + \int_z \int_{\underline{b}(z;\Omega)}^{\infty} y^C(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \tag{B.24}$$

Thus,

$$\begin{aligned}\mathrm{GDP}_{\Omega'} - \mathrm{GDP}_\Omega = &\int_z \int_0^{\underline{b}(z;\Omega')} y^S(b,z;\Omega')h(b,z;\Omega')\mathrm{d}b\mathrm{d}z + \int_z \int_{\underline{b}(z;\Omega')}^{\infty} y^C(b,z;\Omega')h(b,z;\Omega')\mathrm{d}b\mathrm{d}z \\ &- \int_z \int_0^{\underline{b}(z;\Omega)} y^S(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z - \int_z \int_{\underline{b}(z;\Omega)}^{\infty} y^C(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z\end{aligned} \tag{B.25}$$

Because $\Omega' \leq \Omega$, we have $\underline{b}(z;\Omega') \leq \underline{b}(z;\Omega)$, thus

$$\begin{aligned}\mathrm{GDP}_{\Omega'} - \mathrm{GDP}_\Omega = &\int_z \int_{\underline{b}(z;\Omega')}^{\underline{b}(z;\Omega)} \Big[y^C(b,z;\Omega')h(b,z;\Omega') - y^S(b,z;\Omega)h(b,z;\Omega)\Big]\,\mathrm{d}b\mathrm{d}z \\ &+ \int_z \int_{\underline{b}(z;\Omega)}^{\infty} \Big[y^C(b,z;\Omega')h(b,z;\Omega') - y^C(b,z;\Omega)h(b,z;\Omega)\Big]\,\mathrm{d}b\mathrm{d}z \\ &+ \int_z \int_0^{\underline{b}(z;\Omega')} \Big[y^S(b,z;\Omega')h(b,z;\Omega') - y^S(b,z;\Omega)h(b,z;\Omega)\Big]\,\mathrm{d}b\mathrm{d}z.\end{aligned} \tag{B.26}$$

B.5 Proof of Proposition 5

The economy's steady-state aggregate output Y_Ω, capital K_Ω, and labor L_Ω are given by

$$Y_\Omega = \iint_{(b,z)\in\Phi^E} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha}]^{1-\nu} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \tag{B.27}$$

$$K_\Omega = \iint_{(b,z)\in\Phi^E} k(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \tag{B.28}$$

$$L_\Omega = \iint_{(b,z)\in\Phi^E} l(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \tag{B.29}$$

Growth Accounting by Factors Taking logs on both sides of equation (3.6), we obtain

$$\log(\mathrm{TFP}_\Omega) = \log(Y_\Omega) - \alpha\log(K_\Omega) - (1-\alpha)\log(L_\Omega) \tag{B.30}$$

Moving from Ω to Ω', we have a similar expression:

$$\log(\text{TFP}_{\Omega'}) = \log(Y_{\Omega'}) - \alpha \log(K_{\Omega'}) - (1-\alpha)\log(L_{\Omega'}) \tag{B.31}$$

Taking the difference using equations (B.30) and (B.31), we obtain

$$\begin{aligned}\log(\text{TFP}_{\Omega'}) - \log(\text{TFP}_{\Omega}) = & \log(Y_{\Omega'}) - \log(Y_{\Omega}) - \alpha[\log(K_{\Omega'}) - \log(K_{\Omega})] \\ & - (1-\alpha)[\log(L_{\Omega'}) - \log(L_{\Omega})].\end{aligned} \tag{B.32}$$

Equation (B.32) can be approximated by first difference,

$$\frac{\text{TFP}_{\Omega'} - \text{TFP}_{\Omega}}{\text{TFP}_{\Omega}} = \frac{Y_{\Omega'} - Y_{\Omega}}{Y_{\Omega}} - \alpha\frac{K_{\Omega'} - K_{\Omega}}{K_{\Omega}} - (1-\alpha)\frac{L_{\Omega'} - L_{\Omega}}{L_{\Omega}}. \tag{B.33}$$

Let g_x denote the percent change in the value of variable x when the economy moves from Ω to Ω', thus equation (B.33) can be written as

$$g_{TFP} = g_Y - \alpha g_K - (1-\alpha) g_L. \tag{B.34}$$

Since the economy consists of two regimes, aggregate capital and labor are equal to the sum of capital and labor employed by entrepreneurs living in the two regimes separately. Denote $\bar{k}^s_{\Omega}$ and $\bar{l}^s_{\Omega}$ as the average steady-state capital and labor employed by entrepreneurs in the savings regime of the economy Ω:

$$\bar{k}^s_{\Omega} \iint_{(b,z)\in\Phi^S} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z = \iint_{(b,z)\in\Phi^S} k(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \tag{B.35}$$

$$\bar{l}^s_{\Omega} \iint_{(b,z)\in\Phi^S} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z = \iint_{(b,z)\in\Phi^S} l(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \tag{B.36}$$

Denote $\bar{k}^c_{\Omega}$ and $\bar{l}^c_{\Omega}$, as the average steady-state capital and labor employed by entrepreneurs in the credit regime of the economy Ω:

$$\bar{k}^c_{\Omega} \iint_{(b,z)\in\Phi^C} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z = \iint_{(b,z)\in\Phi^C} k(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \tag{B.37}$$

$$\bar{l}^c_{\Omega} \iint_{(b,z)\in\Phi^C} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z = \iint_{(b,z)\in\Phi^C} l(b,z;\Omega)h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \tag{B.38}$$

Denote E_{Ω} as the percent of agents who choose to be entrepreneurs, and p^c_{Ω} as the percent of entrepreneurs living in the credit regime in the steady state of the economy Ω:

$$E_\Omega = \iint_{(b,z)\in\Phi^E} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z \tag{B.39}$$

$$p^c_\Omega E_\Omega = \iint_{(b,z)\in\Phi^C} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \tag{B.40}$$

Thus the economy's steady-state aggregate capital and labor defined in equations (B.28-B.29) can be written as:

$$K_\Omega = E_\Omega(1-p^c_\Omega)\bar{k}^s_\Omega + E_\Omega p^c_\Omega \bar{k}^c_\Omega, \tag{B.41}$$

$$L_\Omega = E_\Omega(1-p^c_\Omega)\bar{l}^s_\Omega + E_\Omega p^c_\Omega \bar{l}^c_\Omega. \tag{B.42}$$

According to equations (B.41-B.42), the growth rates of aggregate capital and labor, g_K and g_L, can be decomposed into:

$$\begin{aligned} g_K &= g_E + (s^c_K - s^s_K)g_{p^c} + s^s_k g_{\bar{k}^s} + s^c_k g_{\bar{k}^c}, \\ g_L &= g_E + (s^c_L - s^s_L)g_{p^c} + s^s_l g_{\bar{l}^s} + s^c_l g_{\bar{l}^c}, \end{aligned} \tag{B.43}$$

where

$$s^c_k = \frac{E_\Omega p^c_\Omega \bar{k}^c_\Omega}{2K_\Omega} + \frac{E_{\Omega'} p^c_{\Omega'} \bar{k}^c_{\Omega'}}{2K_{\Omega'}}, \tag{B.44}$$

$$s^s_k = \frac{E_\Omega (1-p^c_\Omega) \bar{k}^s_\Omega}{2K_\Omega} + \frac{E_{\Omega'} (1-p^c_{\Omega'}) \bar{k}^s_{\Omega'}}{2K_{\Omega'}}, \tag{B.45}$$

$$s^c_l = \frac{E_\Omega p^c_\Omega \bar{l}^c_\Omega}{2L_\Omega} + \frac{E_{\Omega'} p^c_{\Omega'} \bar{l}^c_{\Omega'}}{2L_{\Omega'}}, \tag{B.46}$$

$$s^s_l = \frac{E_\Omega (1-p^c_\Omega) \bar{l}^s_\Omega}{2L_\Omega} + \frac{E_{\Omega'} (1-p^c_{\Omega'}) \bar{l}^s_{\Omega'}}{2L_{\Omega'}}. \tag{B.47}$$

Substituting (B.43) into (B.34), we obtain

$$g_Y = g_{TFP} + g_E + \alpha(s^c_k - s^s_k)g_{p_c} + \alpha s^s_k g_{\bar{k}^s} + \alpha s^c_k g_{\bar{k}^c} + (1-\alpha)(s^c_l - s^s_l)g_{p_c} + (1-\alpha)s^s_l g_{\bar{l}^s} + (1-\alpha)s^c_l g_{\bar{l}^c}. \tag{B.48}$$

Growth Accounting by Regimes The economy's output Y_Ω is equal to the sum of output in each regime,

$$Y_\Omega = E_\Omega(1-p^c_\Omega)\bar{y}^s_\Omega + E_\Omega p^c_\Omega \bar{y}^c_\Omega, \tag{B.49}$$

where $\overline{y}^s_\Omega$ and $\overline{y}^c_\Omega$ are the average output produced by entrepreneurs in the savings and credit regimes:

$$\overline{y}^s_\Omega \iint_{(b,z)\in\Phi^C} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z = \iint_{(b,z)\in\Phi^S} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha}]^{1-\nu} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z, \tag{B.50}$$

$$\overline{y}^c_\Omega \iint_{(b,z)\in\Phi^C} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z = \iint_{(b,z)\in\Phi^C} [z(k(b,z;\Omega)^\alpha l(b,z;\Omega)^{1-\alpha}]^{1-\nu} h(b,z;\Omega)\mathrm{d}b\mathrm{d}z. \tag{B.51}$$

Thus, the growth rate of output can be expressed as

$$g_Y = g_E + (s^c_y - s^s_y)g_{p^c} + s^s_y g_{\overline{y}^s} + s^c_y g_{\overline{y}^c}, \tag{B.52}$$

where

$$s^c_y = \frac{E_\Omega p^c_\Omega \overline{y}^c_\Omega}{2Y_\Omega} + \frac{E_{\Omega'} p^c_{\Omega'} \overline{y}^c_{\Omega'}}{2Y_{\Omega'}}, \tag{B.53}$$

$$s^s_y = \frac{E_\Omega (1-p^c_\Omega)\overline{y}^s_\Omega}{2Y_\Omega} + \frac{E_{\Omega'}(1-p^c_{\Omega'})\overline{y}^s_{\Omega'}}{2Y_{\Omega'}}. \tag{B.54}$$

TFP Decomposition Combining equations (B.34) and (B.52), we obtain

$$\begin{aligned} g_{TFP} = &\left[s^c_y - s^s_y - \alpha(s^c_k - s^s_k) - (1-\alpha)(s^c_l - s^s_l)\right] g_{p^c} \\ &+ s^c_y g_{\overline{y}^c} - \alpha s^c_k g_{\overline{k}^c} - (1-\alpha) s^c_l g_{\overline{l}^c} \\ &+ s^s_y g_{\overline{y}^s} - \alpha s^s_k g_{\overline{k}^s} - (1-\alpha) s^s_l g_{\overline{l}^s}. \end{aligned} \tag{B.55}$$

Thus,

$$\begin{aligned} \mathrm{TFP}_{\Omega'} - \mathrm{TFP}_\Omega = &g_{p^c}\left[s^c_y - s^s_y - \alpha(s^c_k - s^s_k) - (1-\alpha)(s^c_l - s^s_l)\right] \mathrm{TFP}_\Omega \\ &+ \left[s^c_y g_{\overline{y}^c} - \alpha s^c_k g_{\overline{k}^c} - (1-\alpha) s^c_l g_{\overline{l}^c}\right] \mathrm{TFP}_\Omega \\ &+ \left[s^s_y g_{\overline{y}^s} - \alpha s^s_k g_{\overline{k}^s} - (1-\alpha) s^s_l g_{\overline{l}^s}\right] \mathrm{TFP}_\Omega. \end{aligned} \tag{B.56}$$

C Numerical Algorithm

We allow wealth b to vary from 0 to 5, and verify that further increasing wealth does not affect the simulation results. To ensure accuracy, we use 1,000 grids with equal length to discretize the support of wealth when approximating the value functions. We use 100,000 grids to discretize the support of wealth when running simulations. The values between wealth grids are approximated by linear interpolation. We allow productivity z to vary from 1 to the value corresponding to 99.95% of the cumulative distribution function. We use 50 grids with equal length to discretize the support of productivity. We use the golden section search method to find the optimal decision rules. The advantage of the golden section search method is that it is robust to the choice of initial values because convergence is guaranteed. The numerical algorithm is implemented using C++. The program is run on the server of MIT Economics Department, supply.mit.edu, which is built on Dell PowerEdge R910 running RedHat 6.7 (64-core processor, Intel(R) Xeon(R) CPU E7-4870, 2.4GHz). We use OpenMP for parallelization when iterating value functions and simulating the model.

Computing the Steady State

(0). Starting from some arbitrary distribution $h_0(b,z)$.

(1). Guess the wage in the steady state, w.

(2). Guess the interest rate in the steady state, r.

(3). Given the interest rate r and wage w, solve the agent's problem (2.5-2.9) and obtain optimal policies $c(b,z;w,r)$, $k(b,z;w,r)$, $l(b,z;w,r)$ as well as the set of agents who choose to be entrepreneurs $\Phi^E(w,r)$ and the set of entrepreneurs who choose to enter in the credit regime $\Phi^C(w,r)$.

(4). Forward simulate the model by T periods using the optimal policy functions. We set $T = 100$, which sufficiently longer to ensure that the economy reaches the steady-state. Calculate the steady-state joint distribution of wealth and productivity $h_T(b,z)$ according to equation (2.12).

(5). Check the capital market clearing condition (2.10) in period T. If there is excess capital demand (supply), choose a new interest rate r that is greater (smaller) than r and return to step (3). We use bi-section search to form the new guess.

(6). Check the labor market clearing condition (2.11) in period T. If there is excess labor demand (supply), choose a new wage w that is greater (smaller) than w and return to step (2).

Computing the Transitional Dynamics

(0). Solve the steady-state distribution before financial liberalization, $h^{pre}(b,z)$, and the steady-state distribution after financial liberalization, $h^{post}(b,z)$.

(1). Start from the steady-state distribution before financial liberalization, i.e. $h_0(b,z) = h^{pre}(b,z)$.

(2). Guess the wage path $\{w_t\}_{t=0}^{T}$, where T is set sufficiently longer to ensure that the economy can reach the steady-state after financial liberalization.

(3). Guess the interest rate path $\{r_t\}_{t=0}^{T}$.

(4). Taking the wage path $\{w_t\}_{t=0}^{T}$ and interest rate path $\{r_t\}_{t=0}^{T}$ as given, solve the agent's problem (2.5-2.9) for $t = T, ..., 0$ using backward induction, starting from $t = T$. We obtain optimal policies $\{c_t(b,z;w,r), k_t(b,z;w,r), l_t(b,z;w,r)\}_{t=0}^{T}$ as well as the set of agents who choose to be entrepreneurs $\{\Phi_t^E(w,r)\}_{t=0}^{T}$ and the set of entrepreneurs who choose to enter in the credit regime $\{\Phi^C(w,r)\}_{t=0}^{T}$.

(5). Forward simulate the model by T periods using the optimal policy functions, starting from $t = 0$. Calculate the joint distribution of wealth and productivity $\{h_t(b,z)\}_{t=0}^{T}$ according to equation (2.12).

(6). For each $t = 0, 1, ..., T$, holding $h_t(b,z)$ and w_t constant, find the implied interest rate $\tilde{r}_t$ that clears the capital market (2.10).

(7). Calculate $\text{diff}_r = \max\{|r_t - \tilde{r}_t|, \text{for } t = 0, 1, ..., T\}$. If $\text{diff}_r > 10^{-5}$, replace r_t with $(r_t + \tilde{r}_t)/2$ for $t = 0, 1, ..., T$ and return to step (4).

(8). For each $t = 0, 1, ..., T$, holding $h_t(b,z)$ and r_t constant, find the implied wage $\tilde{w}_t$ that clears the labor market (2.11).

(9). Calculate $\text{diff}_w = \max\{|w_t - \tilde{w}_t|, \text{for } t = 0, 1, ..., T\}$. If $\text{diff}_w > 10^{-5}$, replace w_t with $(w_t + \tilde{w}_t)/2$ for $t = 0, 1, ..., T$ and return to step (3).