Online Appendix for "Customer Capital, Talents and Stock Returns"

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Contents

A Micro Foundation for Customer Capital

We use competitive search (see Moen, 1997; Gourio and Rudanko, 2014) to micro found the creation and maintenance of customer capital. The firm's existing customers B_t can purchase goods directly while new customers have to incur flow search costs $x\mathrm{d}t$ before meeting with the firm's sales representatives. Following Gourio and Rudanko (2014), we assume that each agent has constant willingness to pay, denoted as u, and that the firm cannot commit to future product prices. Thus, the firm charges constant price u to existing customers to fully exploit their consumer surplus. The firm offers initial discounts $\tau_t \in [0, \overline{\tau}]$ to attract new customers. In other words, the price is $u - \tau_t$ over $[t, t + \mathrm{d}t]$ for the agents not in B_t. The upper bound $\overline{\tau}$ for initial discounts ensures that the price is at least as high as the average cost per unit of goods.

In the following, we describe the firm's selling problem, the consumer's buying problem, the equilibrium matching, and customer capital growth.

The Firm's Selling Problem. The firm hires sales representatives to build new customer capital. The cost of hiring s_t units of sales representatives over $[t, t + \mathrm{d}t]$ is $\phi(s_t)T_t\mathrm{d}t$ with

$$\phi(s_t) \equiv \alpha s_t^{\eta}, \quad \text{with} \;\; \alpha > 0 \text{ and } \eta > 1. \tag{A.1}$$

The specification of an increasing and convex hiring cost function follows Gourio and Rudanko (2014), which guarantees a decreasing-return-to-scale profit function for hiring sales representatives. By modeling the hiring cost proportional to T_t, we ensure that the firm does not grow out of the cost. We assume that each sales representative has search efficiency T_t to capture the idea that key talents (whose importance is reflected by the value of T_t) are important in bringing new customers. Thus, the firm's effective number of sales representatives is $s_t T_t \mathrm{d}t$ over $[t, t + \mathrm{d}t]$.

Agents' Buying Problem. Agents are aware of the discounts τ_t offered by all firms and decide where to direct their search for goods. Denote $b(\tau_t, s_t; T_t)\mathrm{d}t$ as the number of agents who plan to shop at the firm over $[t, t + \mathrm{d}t]$. Purchases are made when agents meet with the firm's sales representatives. However, due to search and matching frictions, meetings happen with some probability $\lambda(\theta_t)$ depending on the firm's market tightness θ_t:

$$\theta_t = \frac{s_t T_t}{b(\tau_t, s_t; T_t)}. \tag{A.2}$$

From agents' perspective, a tighter market is associated with a greater chance of meeting with the firm's sales representatives. Assuming a Cobb-Douglas matching function, we can derive $\lambda(\theta_t)$ as:

$$\lambda(\theta_t) = \left(\overline{\psi}\theta_t\right)^{1/\chi}, \tag{A.3}$$

where $\overline{\psi} > 0$ denotes the matching efficiency and $\chi > 1$ denotes the matching elasticity.

Equilibrium Matching. The market tightness θ_t is pinned down by the free entry condition. The firm's existing customers B_t have two options. They can either purchase the firm's goods at price u and obtain zero consumer surplus, or they can incur the flow search costs $x\mathrm{d}t$ to purchase other firms' goods with initial discounts τ_t and probability $\lambda(\theta_t)$. In the latter case, the expected consumer surplus net of search costs is $[\tau_t \lambda(\theta_t) - x]\,\mathrm{d}t$. In equilibrium, we have

$$[\tau_t \lambda(\theta_t) - x]\mathrm{d}t = 0. \tag{A.4}$$

Intuitively, this is because the firm offering greater discounts or hiring more sales representatives will attract more potential buyers. The free entry condition ensures that the firm-specific market tightness will adjust until the expected consumer surplus is equalized across all firms. As a result, in equilibrium, agents are indifferent about where to purchase goods. In particular, the firm's existing customers have no incentive to purchase goods from other firms, implying that the customer relationship is long-term in nature.[1]

Substituting equations (A.2) and (A.3) into equation (A.4), we obtain

$$b(\tau_t, s_t; T_t) = \overline{\psi}\tau_t^{\chi} s_t T_t. \tag{A.5}$$

The number of agents meeting with the firm's sales representatives is $b(\tau_t, s_t; T_t)\lambda(\theta_t)\mathrm{d}t$ over $[t, t + \mathrm{d}t]$. Thus, the flow rate of new customers per unit of T_t is

$$\mu(\tau_t, s_t) = \overline{\psi}\tau_t^{\chi-1} s_t. \tag{A.6}$$

Equation (A.6) implies that offering greater discounts and hiring more sales representatives increase the flow rate of new customers, increasing future profits. However, the firm has to pay the hiring cost $\phi(s_t)$ at present, which is costly when the firm's current marginal value of liquidity is high. Therefore, the optimal hiring decision crucially depends on the firm's cash holdings W_t. On the other hand, optimal discounts are trivially set at the upper bound, $\tau_t = \overline{\tau}$, to maximize the flow rate of new customers. This is because discounts are only offered to new customers $\mu(\tau_t, s_t)T_t\mathrm{d}t$ for the initial instant $\mathrm{d}t$. The loss of revenue due to offering greater discounts is of second order.

Thus what matters for the flow rate of new customers is the effective matching efficiency, ψ, defined as

$$\psi \equiv \overline{\psi}\overline{\tau}^{\chi-1}. \tag{A.7}$$

B Numerical Algorithm

The coupled PDEs involve free boundary conditions as the dividend payout boundary, the financing boundary, and the turnover boundary are endogenous. We convert PDEs in continu-

[1]The sticky customer base endows the firm with pricing power, and it has been well recognized in the macroeconomics and industrial organization literature as an important source of imperfect competition (see, e.g. Phelps and Winter, 1970; Rotemberg and Woodford, 1991; Klemperer, 1995; Ravn, Schmitt-Grohe and Uribe, 2006; Gourio and Rudanko, 2014; Gilchrist et al., 2017).

ous time to recursive formulations in discrete time, and implement a dynamic programming algorithm to solve the model.

B.1 Discretization of the Original Problem

Let Δ be the unit of time grid. To formulate the recursive problem, we assume that decisions in period t are made after the realization of lumpy capital shocks $\mathrm{d}M_t$ but before the realization of shocks $a_{t+\Delta}$, $f_{t+\Delta}$, $\mathrm{d}Z_t^c$, and $\xi_{t+\Delta}$. As long as the time grid is sufficiently small, whether decisions are made before or after shock realization would not affect the results. We adopt this timing assumption because (1). it ensures that the firm can issue equity immediately after the realization of lumpy cash flow shocks to avoid dealing with the case of negative cash holdings; (2). it ensures that each state corresponds to a specific set of decisions that are independent of the realized shocks $a_{t+\Delta}$, $f_{t+\Delta}$, $\mathrm{d}Z_t^c$, and $\xi_{t+\Delta}$. See Figure B.1 for the detailed timing of events.

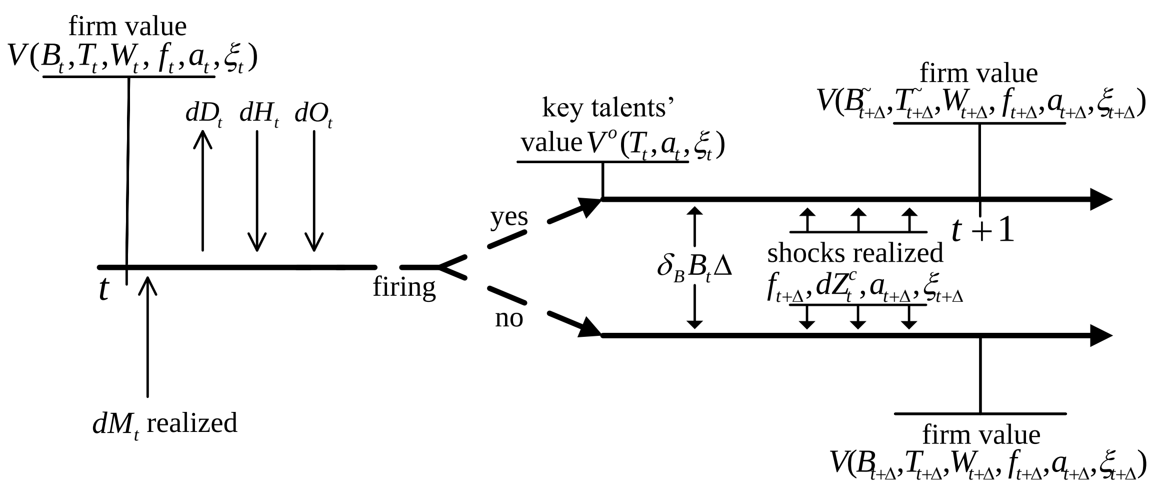

Figure B.1: Timing of events.

The firm solves the following recursive problem in discrete time

$$
\begin{aligned}
&V(B_t, T_t, W_t, f_t, a_t, \xi_t) = \max_{\tau_t, s_t, \mathrm{d}D_t, \mathrm{d}H_t, \overline{\tau_t}, \overline{s_t}, \overline{\mathrm{d}D_t}, \overline{\mathrm{d}H_t}} \\
&(1-\xi_t\Delta)\left[\mathrm{d}D_t - \mathrm{d}H_t - \mathrm{d}X_t + (1-\vartheta\Delta)\mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}V(B_{t+\Delta}, T_{t+\Delta}, W_{t+\Delta}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right)\right. \\
&+\vartheta\Delta\max\left\{\mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}V(B_{t+\Delta}, T_{t+\Delta}, W_{t+\Delta}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right),\right. \\
&\left.\left.\mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}V(B^{\sim}_{t+\Delta}, T^{\sim}_{t+\Delta}, W_{t+\Delta}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right)\right\}\right] \\
&\xi_t\Delta\left[\overline{\mathrm{d}D_t} - \overline{\mathrm{d}H_t} - \overline{\mathrm{d}X_t} + (1-\vartheta\Delta)\mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}V(\overline{B_{t+\Delta}}, \overline{T_{t+\Delta}}, \overline{W_{t+\Delta}}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right)\right. \\
&+\vartheta\Delta\max\left\{\mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}V(\overline{B_{t+\Delta}}, \overline{T_{t+\Delta}}, \overline{W_{t+\Delta}}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right),\right. \\
&\left.\left.\mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}V(\overline{B^{\sim}_{t+\Delta}}, \overline{T^{\sim}_{t+\Delta}}, \overline{W_{t+\Delta}}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right)\right\}\right].
\end{aligned}
\tag{B.1}
$$

In the objective function, shareholders' consumption in the current period is given by dividend $\mathrm{d}D_t$ net of equity issuance $\mathrm{d}H_t$ and issuance costs $\mathrm{d}X_t$. With probability $1-\vartheta\Delta$, the replacement shock does not arrive, in which case the continuation value is $V(B_{t+\Delta}, T_{t+\Delta}, W_{t+\Delta}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})$. With probability $\vartheta\Delta$, the replacement shock arrives, in which case the firm optimally decides whether to fire key talents. The continuation value of firing key talents is given by $V(B^{\sim}_{t+\Delta}, T^{\sim}_{t+\Delta}, W_{t+\Delta}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})$. Expectation is taken with respect to financial constraints risk shocks $\xi_{t+\Delta}$, aggregate productivity shock $a_{t+\Delta}$, customer capital transformation shocks f_t, and cash flow shocks $\mathrm{d}Z^c_t$. As the firm makes decisions after the realization of lumpy cash flow shocks $\mathrm{d}M_t$, there are two cases happening with probabilities $1-\xi_t\Delta$ and $\xi_t\Delta$.

The budget constraint is given by

$$
\begin{aligned}
W_{t+\Delta} =&(1+r\Delta-\rho\Delta)W_t - \mathrm{d}D_t + \mathrm{d}H_t + uB_t\Delta + \sigma_c B_t \mathrm{d}Z^c_t - \phi(s_t)T_t\Delta - \Gamma_t\Delta \\
&- \frac{B_t}{e^{a_t}}\left[\mu(\tau_t, s_t)m_t - \delta_B + \delta_K + \mu_a(a_t-\overline{a}) + \frac{1}{2}\sigma_a^2 a_t\right]\Delta + \frac{\sigma_a\sqrt{a_t}B_t\mathrm{d}Z^a_t}{e^{a_t}},
\end{aligned}
\tag{B.2}
$$

$$
\begin{aligned}
\overline{W_{t+\Delta}} =&(1+r\Delta-\rho\Delta)W_t - \overline{\mathrm{d}D_t} + \overline{\mathrm{d}H_t} + uB_t\Delta + \sigma_c B_t \mathrm{d}Z^c_t - \varsigma B_t - \phi(\overline{s_t})T_t\Delta - \overline{\Gamma_t}\Delta \\
&- \frac{B_t}{e^{a_t}}\left[\mu(\overline{\tau_t}, \overline{s_t})m_t - \delta_B + \delta_K + \mu_a(a_t-\overline{a}) + \frac{1}{2}\sigma_a^2 a_t\right]\Delta + \frac{\sigma_a\sqrt{a_t}B_t\mathrm{d}Z^a_t}{e^{a_t}},
\end{aligned}
\tag{B.3}
$$

If the firm does not fire key talents, the evolution of talent-based customer capital is

$$T_{t+\Delta} = (1 - \delta_B \Delta) T_t + f_t \mu(\tau_t, s_t) T_t \Delta. \tag{B.4}$$

$$\overline{T_{t+\Delta}} = (1 - \delta_B \Delta) T_t + f_t \mu(\overline{\tau_t}, \overline{s_t}) T_t \Delta. \tag{B.5}$$

The evolution of customer capital is given by

$$B_{t+\Delta} = (1 - \delta_B \Delta) B_t + \mu(\tau_t, s_t) T_t \Delta. \tag{B.6}$$

$$\overline{B_{t+\Delta}} = (1 - \delta_B \Delta) B_t + \mu(\overline{\tau_t}, \overline{s_t}) T_t \Delta. \tag{B.7}$$

If the firm fires key talents successfully, the current talent-based customer capital is reduced by a fraction ω. and the next-period talent-based customer capital is given by

$$T^{\sim}_{t+\Delta} = (1 - \omega)(1 - \delta_B \Delta) T_t + f_t \mu(\tau_t, s_t) T_t \Delta. \tag{B.8}$$

$$\overline{T^{\sim}_{t+\Delta}} = (1 - \omega)(1 - \delta_B \Delta) T_t + f_t \mu(\overline{\tau_t}, \overline{s_t}) T_t \Delta. \tag{B.9}$$

The evolution of customer capital is given by

$$B^{\sim}_{t+\Delta} = (1 - \delta_B \Delta) B_t - \omega(1 - \delta_B \Delta) T_t + \mu(\tau_t, s_t) T_t \Delta. \tag{B.10}$$

$$\overline{B^{\sim}_{t+\Delta}} = (1 - \delta_B \Delta) B_t - \omega(1 - \delta_B \Delta) T_t + \mu(\overline{\tau_t}, \overline{s_t}) T_t \Delta. \tag{B.11}$$

The compensation to key talents is determined to honor the continuation value of key talents

$$(\Gamma_t + h B_t)\Delta = V^o(T_t, a_t, \xi_t) - \mathbb{E}_t \left[\frac{\Lambda_{t+\Delta}}{\Lambda_t} V^o(T_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta}) \right]. \tag{B.12}$$

$$(\overline{\Gamma_t} + b B_t)\Delta = V^o(T_t, a_t, \xi_t) - \mathbb{E}_t \left[\frac{\Lambda_{t+\Delta}}{\Lambda_t} V^o(\overline{T_{t+\Delta}}, a_{t+\Delta}, \xi_{t+\Delta}) \right]. \tag{B.13}$$

In addition, the firm's decisions are constrained by

$$\tau_t, \overline{\tau_t} \in [0, \overline{\tau}]; \quad s_t, \mathrm{d}D_t, \mathrm{d}H_t, \overline{s_t}, \overline{\mathrm{d}D_t}, \overline{\mathrm{d}H_t}, \in [0, \infty). \tag{B.14}$$

B.2 Normalized Problem

Because the model is homogeneous of degree zero with respect to the firm's customer capital. We normalize the firm's problem by customer capital B_t to eliminate one state variable. Let $w_t = \frac{W_t}{B_t}$, $m_t = \frac{T_t}{B_t}$, , $\mathrm{d}\tilde{D}_t = \frac{\mathrm{d}D_t}{B_t}$, $\mathrm{d}\tilde{H}_t = \frac{\mathrm{d}H_t}{B_t}$, $\mathrm{d}\tilde{X}_t = \frac{\mathrm{d}X_t}{B_t} = [\gamma + \varpi v^o(m_t, a_t, \xi_t)] \mathbb{1}_{\mathrm{d}\tilde{H}_t > 0} + \varphi \mathrm{d}\tilde{H}_t$, and $\tilde{\Gamma}_t = \frac{\Gamma_t}{B_t}$.

The new state variables are m_t, w_t, f_t, a_t, ξ_t. Let $v(m_t, w_t, f_t, a_t, \xi_t)$ denote the normalized firm value, thus $v(m_t, w_t, f_t, a_t, \xi_t) = \frac{V(B_t, T_t, W_t, f_t, a_t, \xi_t)}{B_t}$.

The normalized firm value is derived from

$$
\begin{aligned}
& v(m_t, w_t, f_t, a_t, \xi_t) = \max_{\tau_t, s_t, \mathrm{d}\tilde{D}_t, \mathrm{d}\tilde{H}_t, \overline{\tau_t}, \overline{s_t}, \overline{\mathrm{d}\tilde{D}_t}, \overline{\mathrm{d}\tilde{H}_t}} \\
& (1 - \xi_t \Delta) \Bigg[\mathrm{d}\tilde{D}_t - \mathrm{d}\tilde{H}_t - [\gamma + \varpi v^o(m_t, a_t, \xi_t)] \mathbb{1}_{\mathrm{d}\tilde{H}_t > 0} - \varphi \mathrm{d}\tilde{H}_t \\
& + (1 - \vartheta\Delta) \mathbb{E}_t \left(\frac{\Lambda_{t+\Delta}}{\Lambda_t} \left(1 - \delta_B \Delta + \mu(\tau_t, s_t) m_t \Delta\right) v(m_{t+\Delta}, w_{t+\Delta}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta}) \right) \\
& + \vartheta\Delta \max \Bigg\{ \mathbb{E}_t \left(\frac{\Lambda_{t+\Delta}}{\Lambda_t} \left(1 - \delta_B \Delta + \mu(\tau_t, s_t) m_t \Delta\right) v(m_{t+\Delta}, w_{t+\Delta}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta}) \right), \\
& \mathbb{E}_t \left(\frac{\Lambda_{t+\Delta}}{\Lambda_t} \left[(1 - \omega m_t)(1 - \delta_B \Delta) + \mu(\tau_t, s_t) m_t \Delta\right] v(m^{\sim}_{t+\Delta}, w^{\sim}_{t+\Delta}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta}) \right) \Bigg\} \Bigg] \\
& \xi_t \Delta \Bigg[\overline{\mathrm{d}\tilde{D}_t} - \overline{\mathrm{d}\tilde{H}_t} - [\gamma + \varpi v^o(m_t, a_t, \xi_t)] \mathbb{1}_{\overline{\mathrm{d}\tilde{H}_t} > 0} - \varphi \overline{\mathrm{d}\tilde{H}_t} \\
& + (1 - \vartheta\Delta) \mathbb{E}_t \left(\frac{\Lambda_{t+\Delta}}{\Lambda_t} \left(1 - \delta_B \Delta + \mu(\overline{\tau_t}, \overline{s_t}) m_t \Delta\right) v(\overline{m_{t+\Delta}}, \overline{w_{t+\Delta}}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta}) \right) \\
& + \vartheta\Delta \max \Bigg\{ \mathbb{E}_t \left(\frac{\Lambda_{t+\Delta}}{\Lambda_t} \left(1 - \delta_B \Delta + \mu(\overline{\tau_t}, \overline{s_t}) m_t \Delta\right) v(\overline{m_{t+\Delta}}, \overline{w_{t+\Delta}}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta}) \right), \\
& \mathbb{E}_t \left(\frac{\Lambda_{t+\Delta}}{\Lambda_t} \left[(1 - \omega m_t)(1 - \delta_B \Delta) + \mu(\overline{\tau_t}, \overline{s_t}) m_t \Delta\right] v(\overline{m^{\sim}_{t+\Delta}}, \overline{w^{\sim}_{t+\Delta}}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta}) \right) \Bigg\} \Bigg],
\end{aligned}
\tag{B.15}
$$

subject to the budget constraints,

$$
\begin{aligned}
& \left[1 - \delta_B \Delta + \mu(\tau_t, s_t) m_t \Delta\right] w_{t+\Delta} = (1 + r\Delta - \rho\Delta) w_t - \mathrm{d}\tilde{D}_t + \mathrm{d}\tilde{H}_t + u\Delta + \sigma_c \mathrm{d}Z_t^c \\
& - \alpha s_t^{\eta} m_t \Delta - \tilde{\Gamma}_t \Delta - \frac{1}{e^{a_t}} \left[\mu(\tau_t, s_t) m_t - \delta_B + \delta_K + \mu_a(a_t - \overline{a}) + \frac{1}{2}\sigma_a^2 a_t \right] \Delta + \frac{\sigma_a \sqrt{a_t} \mathrm{d}Z_t^a}{e^{a_t}},
\end{aligned}
\tag{B.16}
$$

$$
\begin{aligned}
& \left[1 - \delta_B \Delta + \mu(\overline{\tau_t}, \overline{s_t}) m_t \Delta\right] \overline{w_{t+\Delta}} = (1 + r\Delta - \rho\Delta) w_t - \overline{\mathrm{d}\tilde{D}_t} + \overline{\mathrm{d}\tilde{H}_t} + u\Delta + \sigma_c \mathrm{d}Z_t^c - \varsigma \\
& - \alpha \overline{s_t}^{\eta} m_t \Delta - \overline{\tilde{\Gamma}}_t \Delta - \frac{1}{e^{a_t}} \left[\mu(\overline{\tau_t}, \overline{s_t}) m_t - \delta_B + \delta_K + \mu_a(a_t - \overline{a}) + \frac{1}{2}\sigma_a^2 a_t \right] \Delta + \frac{\sigma_a \sqrt{a_t} \mathrm{d}Z_t^a}{e^{a_t}},
\end{aligned}
\tag{B.17}
$$

$$
\begin{aligned}
& \left[(1 - \omega m_t)(1 - \delta_B \Delta) + \mu(\tau_t, s_t) m_t \Delta\right] w^{\sim}_{t+\Delta} = (1 + r\Delta - \rho\Delta) w_t - \mathrm{d}\tilde{D}_t + \mathrm{d}\tilde{H}_t + u\Delta + \sigma_c \mathrm{d}Z_t^c \\
& - \alpha s_t^{\eta} m_t \Delta - \tilde{\Gamma}_t \Delta - \frac{1}{e^{a_t}} \left[\mu(\tau_t, s_t) m_t - \delta_B + \delta_K + \mu_a(a_t - \overline{a}) + \frac{1}{2}\sigma_a^2 a_t \right] \Delta + \frac{\sigma_a \sqrt{a_t} \mathrm{d}Z_t^a}{e^{a_t}},
\end{aligned}
\tag{B.18}
$$

$$
\begin{aligned}
&[(1-\omega m_t)(1-\delta_B\Delta)+\mu(\tau_t,s_t)m_t\Delta]\,\overline{w^{\sim}_{t+\Delta}} = (1+r\Delta-\rho\Delta)w_t - \overline{\mathrm{d}\tilde{D}_t} + \overline{\mathrm{d}\tilde{H}_t} + u\Delta + \sigma_c \mathrm{d}Z^c_t - \varsigma \\
&-\alpha\overline{s_t}^{\eta} m_t\Delta - \overline{\tilde{\Gamma}}_t\Delta - \frac{1}{e^{a_t}}\left[\mu(\overline{\tau_t},\overline{s_t})m_t - \delta_B + \delta_K + \mu_a(a_t-\bar{a}) + \frac{1}{2}\sigma_a^2 a_t\right]\Delta + \frac{\sigma_a\sqrt{a_t}\mathrm{d}Z^a_t}{e^{a_t}}, \qquad \text{(B.19)}
\end{aligned}
$$

the evolution of m_t,

$$
m_{t+\Delta} = \frac{(1-\delta_B\Delta)m_t + f_t\mu(\tau_t,s_t)m_t\Delta}{1-\delta_B\Delta+\mu(\tau_t,s_t)m_t\Delta}, \tag{B.20}
$$

$$
\overline{m_{t+\Delta}} = \frac{(1-\delta_B\Delta)m_t + f_t\mu(\overline{\tau_t},\overline{s_t})m_t\Delta}{1-\delta_B\Delta+\mu(\overline{\tau_t},\overline{s_t})m_t\Delta}, \tag{B.21}
$$

$$
m^{\sim}_{t+\Delta} = \frac{(1-\omega)(1-\delta_B\Delta)m_t + f_t\mu(\tau_t,s_t)m_t\Delta}{(1-\omega m_t)(1-\delta_B\Delta)+\mu(\tau_t,s_t)m_t\Delta}, \tag{B.22}
$$

$$
\overline{m^{\sim}_{t+\Delta}} = \frac{(1-\omega)(1-\delta_B\Delta)m_t + f_t\mu(\overline{\tau_t},\overline{s_t})m_t\Delta}{(1-\omega m_t)(1-\delta_B\Delta)+\mu(\overline{\tau_t},\overline{s_t})m_t\Delta}, \tag{B.23}
$$

the compensation to key talents,

$$
(\tilde{\Gamma}_t + h)\Delta = v^o(m_t,a_t,\xi_t) - \mathbb{E}_t\left[\frac{\Lambda_{t+\Delta}}{\Lambda_t}\left(1-\delta_B\Delta+\mu(\tau_t,s_t)m_t\Delta\right)v^o(m_{t+\Delta},a_{t+\Delta},\xi_{t+\Delta})\right], \tag{B.24}
$$

$$
(\overline{\tilde{\Gamma}}_t + h)\Delta = v^o(m_t,a_t,\xi_t) - \mathbb{E}_t\left[\frac{\Lambda_{t+\Delta}}{\Lambda_t}\left(1-\delta_B\Delta+\mu(\overline{\tau_t},\overline{s_t})m_t\Delta\right)v^o(\overline{m_{t+\Delta}},a_{t+\Delta},\xi_{t+\Delta})\right], \tag{B.25}
$$

The normalized continuation value $v^o(m_t,a_t,\xi_t) = \frac{V^o(T_t,a_t,\xi_t)}{B_t}$ is given by

$$
v^o(m_t,a_t,\xi_t) = v^n(m_t,a_t,\xi_t) + \gamma(\omega+\ell)m_t + \varphi W_0^*(\omega+\ell)m_t, \tag{B.26}
$$

where $v^n(m_t,a_t,\xi_t)$ is derived from equation (B.27) and W_0^* is the optimal solution to equation (B.27),

$$
v^n(m_t,a_t,\xi_t) = \max_{W_0}(\omega+\ell)m_t\left[-\gamma-(1+\varphi)W_0 + \mathbb{E}^{\tilde{f}}[v(\tilde{f},W_0,\tilde{f},a_t,\xi_t)]\right]. \tag{B.27}
$$

As we explain in subsection B.3, implementing our numerical algorithms also require solving two special cases of the normalized problem, one with zero financing costs and one with no new customer flows. We write down their formulations below.

Zero Financing Costs When the financing costs are zero, the firm does not hold cash. The firm's state variable are m, f, a, and ξ. Given our calibration, the firm does not fire key talents because the expected cash inflows generated by key talents are larger than their compensation.

Thus the firm solves the following problem to maximize shareholder value:

$$
\begin{aligned}
&v(m_t, f_t, a_t, \xi_t) = \max_{\tau_t, s_t, \overline{\tau_t}, \overline{s_t}} \\
&(1-\xi_t\Delta)\left[u\Delta - \alpha s_t^{\eta} m_t\Delta - \tilde{\Gamma}_t\Delta - \frac{1}{e^{a_t}}\left(\mu(\tau_t, s_t)m_t - \delta_B + \delta_K + \mu_a(a_t - \overline{a}) + \frac{1}{2}\sigma_a^2 a_t\right)\Delta\right. \\
&\left. + \mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}\left(1 - \delta_B\Delta + \mu(\tau_t, s_t)m_t\Delta\right) v(m_{t+\Delta}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right)\right] \\
&+ \xi_t\Delta\left[u\Delta - \varsigma - \alpha\overline{s_t}^{\eta} m_t\Delta - \overline{\tilde{\Gamma}}_t\Delta - \frac{1}{e^{a_t}}\left(\mu(\overline{\tau_t}, \overline{s_t})m_t - \delta_B + \delta_K + \mu_a(a_t - \overline{a}) + \frac{1}{2}\sigma_a^2 a_t\right)\Delta\right. \\
&\left. + \mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}\left(1 - \delta_B\Delta + \mu(\overline{\tau_t}, \overline{s_t})m_t\Delta\right) v(\overline{m_{t+\Delta}}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right)\right],
\end{aligned}
\tag{B.28}
$$

subject to the evolution of m_t,

$$
m_{t+\Delta} = \frac{(1-\delta_B\Delta)m_t + f_t\mu(\tau_t, s_t)m_t\Delta}{1 - \delta_B\Delta + \mu(\tau_t, s_t)m_t\Delta}, \tag{B.29}
$$

$$
\overline{m_{t+\Delta}} = \frac{(1-\delta_B\Delta)m_t + f_t\mu(\overline{\tau_t}, \overline{s_t})m_t\Delta}{1 - \delta_B\Delta + \mu(\overline{\tau_t}, \overline{s_t})m_t\Delta}, \tag{B.30}
$$

the compensation to key talents,

$$
\tilde{\Gamma}_t\Delta = v^o(m_t, a_t, \xi_t) - E_t\left[\frac{\Lambda_{t+\Delta}}{\Lambda_t}\left(1 - \delta_B\Delta + \mu(\tau_t, s_t)m_t\Delta\right) v^o(m_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right], \tag{B.31}
$$

$$
\overline{\tilde{\Gamma}}_t\Delta = v^o(m_t, a_t, \xi_t) - E_t\left[\frac{\Lambda_{t+\Delta}}{\Lambda_t}\left(1 - \delta_B\Delta + \mu(\overline{\tau_t}, \overline{s_t})m_t\Delta\right) v^o(\overline{m_{t+\Delta}}, a_{t+\Delta}, \xi_{t+\Delta})\right], \tag{B.32}
$$

where the normalized continuation value $v^o(m_t, a_t, \xi_t) = \frac{V^o(T_t, a_t, \xi_t)}{B_t}$ is given by

$$
v^o(m_t, a_t, \xi_t) = (\omega + \ell)m_t\mathbb{E}_t^{\tilde{f}}\left[v\left(\tilde{f}, \tilde{f}, a_t, \xi_t\right)\right]. \tag{B.33}
$$

No New Customer Flows When we set $\psi = 0$, there are no new customer flows. Thus the firm's hiring and discount decisions are trivially determined by $\tau_t = s_t = \overline{\tau_t} = \overline{s_t} = 0$. The normalized firm value can be written as

$$
\begin{aligned}
& v(m_t, w_t, f_t, a_t, \xi_t) = \max_{\mathrm{d}\tilde{D}_t, \mathrm{d}\tilde{H}_t, \overline{\mathrm{d}\tilde{D}_t}, \overline{\mathrm{d}\tilde{H}_t}} \\
& (1-\xi_t\Delta)\left[\mathrm{d}\tilde{D}_t - \mathrm{d}\tilde{H}_t - [\gamma + \varpi v^o(m_t, a_t, \xi_t)]\mathbb{1}_{\mathrm{d}\tilde{H}_t>0} - \varphi \mathrm{d}\tilde{H}_t \right. \\
& + (1-\vartheta\Delta)\mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}(1-\delta_B\Delta)\, v(m_{t+\Delta}, w_{t+\Delta}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right) \\
& + \vartheta\Delta \max\left\{\mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}(1-\delta_B\Delta)\, v(m_{t+\Delta}, w_{t+\Delta}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right),\right. \\
& \left.\left.\mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}(1-\delta_B\Delta)(1-\omega m_t) v(m^{\sim}_{t+\Delta}, w^{\sim}_{t+\Delta}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right)\right\}\right] \\
& \xi_t\Delta\left[\overline{\mathrm{d}\tilde{D}_t} - \overline{\mathrm{d}\tilde{H}_t} - [\gamma + \varpi v^o(m_t, a_t, \xi_t)]\mathbb{1}_{\overline{\mathrm{d}\tilde{H}_t}>0} - \varphi \overline{\mathrm{d}\tilde{H}_t} \right. \\
& + (1-\vartheta\Delta)\mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}(1-\delta_B\Delta)\, v(\overline{m_{t+\Delta}}, \overline{w_{t+\Delta}}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right) \\
& + \vartheta\Delta \max\left\{\mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}(1-\delta_B\Delta)\, v(\overline{m_{t+\Delta}}, \overline{w_{t+\Delta}}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right),\right. \\
& \left.\left.\mathbb{E}_t\left(\frac{\Lambda_{t+\Delta}}{\Lambda_t}(1-\delta_B\Delta)(1-\omega m_t) v(\overline{m^{\sim}_{t+\Delta}}, \overline{w^{\sim}_{t+\Delta}}, f_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right)\right\}\right], \qquad \text{(B.34)}
\end{aligned}
$$

subject to the budget constraints,

$$
\begin{aligned}
(1-\delta_B\Delta)w_{t+\Delta} = & (1 + r\Delta - \rho\Delta)w_t - \mathrm{d}\tilde{D}_t + \mathrm{d}\tilde{H}_t + u\Delta + \sigma_c \mathrm{d}Z^c_t - \tilde{\Gamma}_t\Delta \\
& - \frac{1}{e^{a_t}}\left[-\delta_B + \delta_K + \mu_a(a_t - \overline{a}) + \frac{1}{2}\sigma_a^2 a_t\right]\Delta + \frac{\sigma_a\sqrt{a_t}\mathrm{d}Z^a_t}{e^{a_t}}, \qquad \text{(B.35)}
\end{aligned}
$$

$$
\begin{aligned}
(1-\delta_B\Delta)\overline{w_{t+\Delta}} = & (1 + r\Delta - \rho\Delta)w_t - \overline{\mathrm{d}\tilde{D}_t} + \overline{\mathrm{d}\tilde{H}_t} + u\Delta + \sigma_c \mathrm{d}Z^c_t - \overline{\tilde{\Gamma}}_t\Delta - \varsigma \\
& - \frac{1}{e^{a_t}}\left[-\delta_B + \delta_K + \mu_a(a_t - \overline{a}) + \frac{1}{2}\sigma_a^2 a_t\right]\Delta + \frac{\sigma_a\sqrt{a_t}\mathrm{d}Z^a_t}{e^{a_t}}, \qquad \text{(B.36)}
\end{aligned}
$$

$$
\begin{aligned}
(1-\omega m_t)(1-\delta_B\Delta)w^{\sim}_{t+\Delta} = & (1 + r\Delta - \rho\Delta)w_t - \mathrm{d}\tilde{D}_t + \mathrm{d}\tilde{H}_t + u\Delta + \sigma_c \mathrm{d}Z^c_t - \tilde{\Gamma}_t\Delta \\
& - \frac{1}{e^{a_t}}\left[-\delta_B + \delta_K + \mu_a(a_t - \overline{a}) + \frac{1}{2}\sigma_a^2 a_t\right]\Delta + \frac{\sigma_a\sqrt{a_t}\mathrm{d}Z^a_t}{e^{a_t}}, \qquad \text{(B.37)}
\end{aligned}
$$

$$(1-\omega m_t)(1-\delta_B\Delta)\overline{w^{\sim}_{t+\Delta}} =(1+r\Delta-\rho\Delta)w_t - \overline{\mathrm{d}\tilde{D}_t} + \overline{\mathrm{d}\tilde{H}_t} + u\Delta + \sigma_c \mathrm{d}Z^c_t - \overline{\tilde{\Gamma}}_t\Delta - \varsigma$$
$$- \frac{1}{e^{a_t}}\left[-\delta_B + \delta_K + \mu_a(a_t-\bar{a}) + \frac{1}{2}\sigma_a^2 a_t\right]\Delta + \frac{\sigma_a\sqrt{a_t}\mathrm{d}Z^a_t}{e^{a_t}}, \tag{B.38}$$

the evolution of m_t,

$$m_{t+\Delta} = \overline{m_{t+\Delta}} = m_t, \tag{B.39}$$

$$m^{\sim}_{t+\Delta} = \overline{m^{\sim}_{t+\Delta}} = \frac{1-\omega}{1-\omega m_t} m_t, \tag{B.40}$$

the compensation to key talents,

$$\tilde{\Gamma}_t\Delta = v^o(m_t, a_t, \xi_t) - \mathbb{E}_t\left[\frac{\Lambda_{t+\Delta}}{\Lambda_t}\left(1-\delta_B\Delta\right)v^o(m_{t+\Delta}, a_{t+\Delta}, \xi_{t+\Delta})\right], \tag{B.41}$$

$$\overline{\tilde{\Gamma}}_t\Delta = v^o(m_t, a_t, \xi_t) - \mathbb{E}_t\left[\frac{\Lambda_{t+\Delta}}{\Lambda_t}\left(1-\delta_B\Delta\right)v^o(\overline{m_{t+\Delta}}, a_{t+\Delta}, \xi_{t+\Delta})\right], \tag{B.42}$$

The normalized continuation value $v^o(m_t, a_t, \xi_t) = \frac{V^o(T_t, a_t, \xi_t)}{B_t}$ is given by

$$v^o(m_t, a_t, \xi_t) = v^n(m_t, a_t, \xi_t) + \gamma(\omega+\ell)m_t + \varphi W^*_0(\omega+\ell)m_t, \tag{B.43}$$

where $v^n(m_t, a_t, \xi_t)$ is derived from equation (B.44) and W^*_0 is the optimal solution to equation (B.44),

$$v^n(m_t, a_t, \xi_t) = \max_{W_0}(\omega+\ell)m_t\left[-\gamma-(1+\varphi)W_0 + \mathbb{E}^{\tilde{f}}[v\left(\tilde{f}, W_0, \tilde{f}, a_t, \xi_t\right)]\right]. \tag{B.44}$$

B.3 Implementation

We discretize the cash flow shocks based on 11 grids spanning from $-3\sigma_c$ to $3\sigma_c$ and the aggregate productivity shocks based on 5 grids spanning from $-3\sigma_a$ to $3\sigma_a$ using the method of Tauchen (1986). In particular,

$$a_{t+\Delta} = (1-\mu_a\Delta)a_t + \mu_a\bar{a}\Delta + \sigma_a\sqrt{a_t\Delta}z^a, \tag{B.45}$$

where $z^a \sim N(0,1)$. The unconditional standard deviation of a_t can be approximated as

$$\sigma_a\sqrt{\frac{\bar{a}\Delta}{1-(1-\mu_a\Delta)^2}} \tag{B.46}$$

We select the maximum value s_{a_5} as a multiple 3 of the unconditional standard deviation, i.e.

$$s_{a_5} = \overline{a} + 3\sigma_a \sqrt{\frac{\overline{a}\Delta}{1-(1-\mu_a\Delta)^2}} \tag{B.47}$$

Let $s_{a_1} = 2 * \overline{a} - s_{a_5}$ because the unconditional mean of a_t is $\overline{a}$. Let $\{s_{a_2}, s_{a_3}, s_{a_4}\}$ be located in a equispaced manner over the interval $[s_{a_1}, s_{a_5}]$. Denote $d = s_{a_2} - s_{a_1}$ as the distance between successive points in the productivity grid.

The generic transition probability π_{jk} is given by

$$\begin{aligned}
\pi_{jk} =& \Pr\left\{a_{t+1} = s_{a_k} | a_t = s_{a_j}\right\} \\
=& \Pr\left\{s_{a_k} - d/2 < (1-\mu_a\Delta)s_{a_j} + \mu_a\overline{a}\Delta + \sigma_a\sqrt{s_{a_j}\Delta}z^a \leq s_{a_k} + d/2\right\} \\
=& \Pr\left\{\frac{s_{a_k} - d/2 - (1-\mu_a\Delta)s_{a_j} - \mu_a\overline{a}\Delta}{\sigma_a\sqrt{s_{a_j}\Delta}} < z_a \leq \frac{s_{a_k} + d/2 - (1-\mu_a\Delta)s_{a_j} - \mu_a\overline{a}\Delta}{\sigma_a\sqrt{s_{a_j}\Delta}}\right\}
\end{aligned} \tag{B.48}$$

Then, if $1 < k < 5$, for each j choose

$$\pi_{jk} = \Phi\left(\frac{s_{a_k} + d/2 - (1-\mu_a\Delta)s_{a_j} - \mu_a\overline{a}\Delta}{\sigma_a\sqrt{s_{a_j}\Delta}}\right) - \Phi\left(\frac{s_{a_k} - d/2 - (1-\mu_a\Delta)s_{a_j} - \mu_a\overline{a}\Delta}{\sigma_a\sqrt{s_{a_j}\Delta}}\right) \tag{B.49}$$

while for the boundaries of the interval $k = 1$ and $k = 5$, choose

$$\begin{aligned}
\pi_{j1} &= \Phi\left(\frac{s_{a_1} + d/2 - (1-\mu_a\Delta)s_{a_j} - \mu_a\overline{a}\Delta}{\sigma_a\sqrt{s_{a_j}\Delta}}\right) \\
\pi_{j5} &= 1 - \Phi\left(\frac{s_{a_5} - d/2 - (1-\mu_a\Delta)s_{a_j} - \mu_a\overline{a}\Delta}{\sigma_a\sqrt{s_{a_j}\Delta}}\right)
\end{aligned} \tag{B.50}$$

We use collocation methods to solve the firm's problem (B.15). Let $S = S_m \times S_w \times S_f \times S_a \times S_\xi$ be the grid of collocation nodes, where $S_m = \{s_{m_1}, s_{m_2}, ..., s_{m_l}\}$, $S_w = \{s_{w_1}, s_{w_2}, ..., s_{w_k}\}$, $S_f = \{s_{f_1}, s_{f_2}\}$, $S_a = \{s_{a_1}, ..., s_{a_5}\}$ and $S_\xi = \{s_{\xi_1}, s_{\xi_2}\}$. Our solution indicates the model is highly nonlinear in w, thus we set $w_k = 101$ and use 100 equi-spaced nodes from 0 to 0.6 to construct S_w. The lower bound s_{w_1} is set to be zero because we restrict the firm to have non-zero cash holdings in our model. The firm's dividend payout boundary increases with m and the arrival rate of lumpy cash flow shocks. We thus choose the upper bound s_{w_k} so that even when the arrival rate is high (i.e. $\xi = \xi_H$), the dividend payout boundary $\overline{w}$ is below s_{w_k} for the firm with $m = 1$. Our solution indicates that the firm's decisions are approximately linear in m, thus we set $m_l = 11$ and use 10 equi-spaced nodes from 0 to 1 to construct S_m. The states $s_{a_1}, ..., s_{a_5}$

correspond to the five levels of aggregate productivity shocks. The states s_{f_1}, s_{f_2} correspond to the two levels of talent-based transformation rate $f_{(1)}, f_{(2)}$. The states s_{ξ_1} and s_{ξ_2} correspond to the arrival rates of lumpy cash flow shocks, ξ_L and ξ_H. When solving continuous time models in discrete time, it is important to choose the time grid consistent with the state space. This is because if the time grid is too dense relative to the state grid, the solution of the value functions tend to be non-smooth. If the time grid is too sparse relative to the state grid, computing power is wasted without increasing accuracy. By trial and error, we set our time grid $\Delta = 1/365$, which implies that one period in our discretized model represents one day.

We approximate the firm's value function $v(m, w, f, a, \xi)$ on S using a linear spline with coefficients corresponding to each grid point. We first form a guess for the spline's $m_l \times w_k \times 2 \times 5 \times 2$ coefficients, then we iterate to obtain a vector that solves the system of $m_l \times w_k \times 2 \times 5 \times 2$ Bellman equations. Below we detail the procedures of forming initial guess and calculating iterations.

Initial Guess The major difficulty of implementing a dynamic programming algorithm to solve a continuous-time value function is that the convergence of value functions requires tens of thousands of iterations. When solving the model, we find that the marginal value of liquidity, $v_w(m, w, f, a, \xi)$ converges to the true value much faster than the convergence of the value function $v(m, w, f, a, \xi)$. In many macroeconomics and finance models, the absolute value of value functions would not matter for the decisions made by economic agents. However, in our model, the cost of firing key talents is captured by a reduction in the firm's customer capital. In the normalized problem (B.15), the cost of firing key talents increases with the absolute value of $v(m, w, f, a, \xi)$. Therefore, in order to get the firm's firing decisions right, we have to accurately solve the absolute value of $v(m, w, f, a, \xi)$.

In principle, we can start with any initial guess of the value function $v(m, w, f, a, \xi)$ and do a sufficiently large number of iterations to reach convergence, which is guaranteed by the contraction mapping theorem. However, this is very costly as each iteration takes several minutes to finish given that our model has two endogenous state variables and four decision variables. To form an initial guess that is close to the final solution, we solve the model in three step.

First, we solve the firm's problem (B.28) in a perfect financial market. This problem is much easier to solve because m is the only endogenous state variable. We iterate the value functions for about 60,000 times to reach a convergence until the level of accuracy satisfies $max_{(m,f,a,\xi)\in\{S_m \times S_f \times S_a \times S_\xi\}} |V^1_{n^1-1}(m, f, a, \xi) - V^1_{n^1}(m, f, a, \xi)| < 10^{-16}$ at the $n^1 th$ iteration.

Second, we use the value functions $V^1(m, f, a, \xi)$ from the perfect financial market to initialize the firm's value functions in problem (B.34) with no new customer flows. Specifically, we initialize the value of $V^2(m, w, f, a, \xi)$ at collocation nodes $S_m \times S_w \times S_f \times S_a \times S_\xi$ by setting

$V_0^2(m,w,f,a,\xi) = V_{n^1}^1(m,f,a,\xi) + w$. We start with the first iteration with perfect financial market, i.e. $\gamma = \varphi = 0$, and linearly increase the financing costs to the calibrated values of γ and φ in the first 100 iterations. This is to ensure that the value and policy functions move smoothly from the perfect financial market to the frictional financial market. We then continue iterating the value functions $V^2(m,w,f,a,\xi)$ for another 6,000 times to reach a convergence until the level of accuracy satisfies $max_{(m,w,f,a,\xi)\in\{S_m\times S_w\times S_f\times S_a\times S_\xi\}}|V_{n^2-1}^2(m,w,f,a,\xi) - V_{n^2}^2(m,w,f,a,\xi)| < 10^{-7}$ at the n^2th iteration.

Third, we use the value functions $V^2(m,w,f,a,\xi)$ to initialize the firm's value functions in problem (B.15) by setting $V_0^3(m,w,f,a,\xi) = V_{n^2}^2(m,w,f,a,\xi)$. Then we iterate the value functions $V_0^3(m,w,f,a,\xi)$ for about 6,000 times to reach a convergence until the level of accuracy satisfies $max_{(m,w,f,a,\xi)\in\{S_m\times S_w\times S_f\times S_a\times S_\xi\}}|V_{n^3-1}^3(m,w,f,a,\xi) - V_{n^3}^3(m,w,f,a,\xi)| < 10^{-7}$ at the n^3th iteration.

Calculating Iterations Given the value functions from the previous iteration, we use golden section search to find the optimal financing, payout, discounts, and sales decisions. The way to search for the optimal discounts and sales decisions are straight forward because both decision variables are continuous. When searching for the discount decision, we set the initial upper bound to be $\overline{\tau}$ and the the initial lower bound to be 0. When searching for the optimal sales decision, we set the initial lower bound to be zero and the initial upper bound is set to guarantee a non-negative cash ratio in the next period even when the worst idiosyncratic cash flow shock occurs.

Searching for the optimal financing and payout decisions are more involved because they only happen at the boundaries when time is continuous. There are two complications when solving this continuous-time model in discrete time. First, the firm starts to issue equity in advance before cash ratio exactly hits the zero lower bound. Second, the discretized cash flow shocks may drive the next period cash ratio to a negative number even when the firm's current period cash ratio is strictly positive. We deal with the first problem by choosing a fine time grid $\Delta = 1/365$. This ensures a reasonably good approximation as the firm issues equity when cash ratio drops below 0.01, the smallest cash ratio grid we consider. To deal with the second issue, we impose a sufficiently large penalty on the firm's value (in the code, we subtract the firm's value by 10) whenever the next-period cash ratio drops below zero. This is to ensure that the firm would issue equity whenever there is a chance to have a negative cash ratio in the next period given our discretization of cash flow shocks. The large penalty is important to guarantee the convergence of the value functions. If the penalty is not large enough, then the firm may wait for a bit longer before issuing equity, and as a result, the optimal financing boundary is not correctly solved at this level of discretization.

Having dealt with the two issues above, we search for optimal financing and payout boundaries separately using golden section search. Regarding the financing boundary, we set the initial lower bound to be zero and the initial upper bound to be the highest cash ratio grid, s_{w_k}. If the search algorithm returns zero, it means that the firm does not issue equity in that state. Otherwise, the search algorithm returns some positive number, which is the optimal amount of equity issued by the firm. Therefore, the golden section search allows us to simultaneously find the financing boundary and the amount of financing. Regarding the payout boundary, we set the initial lower bound to be zero and the initial upper bound to be the firm's current cash ratio. Then similarly, the golden section search allows us to find both the payout boundary and the amount of dividend.

C Examples of Building and Maintaining Customer Capital

Now, let us elaborate on how key talents and pure brand loyalty create and maintain a firm's customer capital in different ways. Key talents are the essential employees of a firm, mainly including managers and innovators. Managers frequently bring in new businesses and customer relationships through personal connections and specialized skills; meanwhile, innovators in R&D teams often develop products with creative features that can attract new customers. These are typical examples of customer capital growth due to key talents' unique contributions. Managers can also bring in new customers through designing advertisement and marketing campaigns. These are examples of customer capital growth due to the combination of both forces. Moveover, pure brand perception alone can also bring in new customers. For example, consumers sometimes become aware of the firm's products after friends' recommendations based on their pure brand loyalty, which is referred to as *word-of-mouth marketing*. As supported by ample evidence in the marketing literature, the main force of future customer capital growth is key talents' contribution. In addition to creating future customer capital growth, both key talents and pure brand loyalty are also important in maintaining the existing customer relationships. When new customers are brought into the firm, some become part of talent-based customer capital, while others become loyal to the firm's brands. New customers brought by personal connections or innovations, for example, are more likely to become part of talent-based customer capital, compared to new customers brought by advertisements and friends' recommendations.

D BTR and Financial Constraints

Our model implies that there exist nonlinear interactions between BTR and financial constraints. In particular, the effect of BTR is greater among those financial constrained firms. Here, we conduct a split sample analyses based on the measures of financial constraints. We find that the negative relation between BTR and stock returns are more pronounced among financially constrained firms, suggesting that the firms with lower BTRs have larger exposure to financial constraints risk.

Following the literature, we use three measures to capture financial constraints: the HP index (see Hadlock and Pierce, 2010), the WW index (see Whited and Wu, 2006; Hennessy and Whited, 2007), and firm size measured by the market capitalization of equity (see, e.g. Gilchrist and Himmelberg, 1995; Livdan, Sapriza and Zhang, 2009; Hadlock and Pierce, 2010; Li, 2011). The firms with higher HP index, higher WW index, and smaller size are more likely to be financially constrained.

In June of year t, we sort firms into three groups based on their financial constraint measures. We further sort firms in each group into five quintiles based on firms' BTR in year $t-1$. We compute the value-weighted portfolio returns and estimate their alphas using various asset pricing models. Table D.1 presents the average excess returns and alphas of the BMT portfolios. Although the average excess returns and alphas of the BMT portfolios are negative in all groups, the magnitudes of the average excess returns and alphas are much larger among financially constrained firms. This pattern is robust to the choice of financial constraint measures and asset pricing models. These findings suggest that the asset pricing implications of BTR are closely related to firms' financial constraints risk.

E BTR and Tunovers

E.1 BTR and CEO Tunovers

In the main text of our paper, we examine the relation between BTR and executive turnovers. Here, we replicate the analyses by including only CEOs. As explained in the main text, we focus on non-retirement turnovers. We use two approaches to define non-retirement CEO turnovers. The first approach is solely based on the age of CEOs. We follow the literature (see, e.g. Parrino, 1997; Jenter and Kanaan, 2015) and use age 60 as the cutoff for the retirement age.[2] We define CEO turnovers as non-retirement turnovers if CEOs leave their firms at age 59 or younger due to reasons other than death. The indicator variable for non-retirement turnovers for firm i in year t is denoted as $\text{Turnover}_{i,t}^{(1)}$. The second approach uses additional information from Execucomp, which classifies CEO turnovers into four groups: retirement, death, unknown, and resignation. We define CEO turnovers as non-retirement turnovers if CEOs leave their firms at age 59 or younger due to reasons other than death or if CEOs leave their firms due to

[2]Our results are robust to other age cutoffs such as 65.

Table D.1: Excess BMT portfolio returns across subsamples split by financial constraints.

Low HP	Medium HP	High HP	Low WW	Medium WW	High WW	Big Size	Medium Size	Small Size
Panel A: Excess Return (%)								
-2.64	0.21	-8.46^{**}	-1.82	-1.86	-9.79^{**}	-2.11	-6.80^{**}	-8.80^{**}
[-1.48]	[0.07]	[-2.44]	[-0.72]	[-0.62]	[-1.99]	[-0.65]	[-2.05]	[-2.01]
Panel B: Fama-French Three-Factor α (%)								
-3.31^{*}	0.11	-7.51^{**}	-2.46	-2.66	-10.32^{**}	-1.25	-8.40^{***}	-9.89^{**}
[-1.85]	[0.04]	[-2.28]	[-1.05]	[-0.96]	[-2.18]	[-0.47]	[-2.77]	[-2.06]
Panel C: Carhart Four-Factor α (%)								
-3.19^{*}	-0.16	-7.27^{**}	-2.79	-3.24	-10.24^{**}	-2.79	-7.10^{**}	-11.43^{**}
[-1.76]	[-0.05]	[-2.09]	[-1.18]	[-1.15]	[-2.14]	[-1.06]	[-2.34]	[-2.37]
Panel D: Pástor-Stambaugh Five-Factor α (%)								
-2.99	0.52	-7.20^{**}	-2.59	-2.88	-9.32^{*}	-2.59	-6.56^{**}	-11.28^{**}
[-1.64]	[0.17]	[-2.06]	[-1.09]	[-1.02]	[-1.94]	[-0.97]	[-2.16]	[-2.32]
Panel E: Hou-Xue-Zhang q-factors α (%)								
-3.35^{*}	-2.50	-12.09^{***}	-5.47^{**}	-6.21^{**}	-16.13^{***}	-8.05^{***}	-10.29^{***}	-12.86^{**}
[-1.81]	[-0.81]	[-3.00]	[-2.23]	[-2.17]	[-3.18]	[-3.22]	[-3.08]	[-2.53]
Panel F: Fama-French Five-Factor α (%)								
-3.25^{*}	-2.34	-11.49^{***}	-4.56^{*}	-5.62^{**}	-14.03^{***}	-7.16^{***}	-9.55^{***}	-10.85^{**}
[-1.76]	[-0.79]	[-2.96]	[-1.89]	[-1.98]	[-2.87]	[-2.87]	[-3.01]	[-2.17]

Note: This table shows the brand-minus-talent (BMT) portfolio returns across subsamples split by financial constraints. In June of year t, we sort firms into three groups based on the finanical constrain measures: the HP index (see Hadlock and Pierce, 2010), the WW index (see Whited and Wu, 2006; Hennessy and Whited, 2007), and the firm size measured by the market capitalization of equity. We then sort firms in each group into five quintiles based on firms' BTR in year $t-1$. Once the portfolios are formed, their monthly returns are tracked from July of year t to June of year $t+1$. BTR is the ratio between brand stature and brand strength. Brand stature and brand strength are two brand metrics constructed by the BAV Group based on its comprehensive brand perception survey. Brand stature measures brand loyalty of existing customers. Brand strength measures how much the brand is perceived by the consumers to be innovative and distinctive. We compute the value-weighted portfolio returns and report the average excess returns of the long/short BTR portfolio, which is deonoted as the brand-minus-talent (BMT) portfolio. We also report the portfolio alphas estimated by the Fama-French three-factor model, the Carhart four-factor model, the Pástor-Stambaugh five-factor model, the Hou-Xue-Zhang q-factors model and the Fama-French five-factor model. Our sample includes firms that are listed on NYSE, AMEX, and NASDAQ exchanges with share codes 10 or 11. We exclude financial firms and utility firms from the analysis. The sample period is 1993 to 2016. We include t-statistics in parentheses. Standard errors are computed using the Newey-West estimator allowing for one lag of serial correlation in returns. We annualize the average excess returns and the alphas by multiplying by 12. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

resignations according to the Execucomp data. The indicator variable for the non-retirement turnovers for firm i in year t is denoted as $\text{Turnover}_{i,t}^{(2)}$.

We run the following regression to study the relation between BTR and CEO turnovers:

$$\text{Turnover}_{i,t} \times 100 = \alpha_{ind} + \alpha_t + \beta ln\text{BTR}_{i,t-1} + \gamma'\text{Controls}_{i,t-1} + \varepsilon_{i,t}. \tag{E.1}$$

Table E.2 tabulates the regression results. We find that non-retirement CEO turnover rates are significantly lower in the firms with higher BTRs. This result is robust to the two definitions of the non-retirement turnovers, and it is also robust to the inclusion of the SIC-2 industry fixed effects. The negative relation between BTR and CEO turnovers is economically significant. According to the specification with both SIC-2 industry fixed effects and year fixed effects, a one standard deviation increase in lnBTR leads to a decrease in the probability of the non-retirement CEO turnovers by 0.902 percentage point, which is roughly 1/5 of the average non-retirement turnover rate in the data.

The time-series variation of the BMT (i.e., brand-minus-talent) returns captures the changes in the aggregate funding liquidity conditions. Our model predicts that the key talent turnover rates of low BTR firms are higher due to their higher levels of operating leverage. The difference in the turnover rates should be stronger when firms face adverse aggregate funding liquidity conditions. To test this prediction, we interact the standardize lnBTR with BMT and include the interaction term as the main independent variable in the following regressions:

$$\text{Turnover}_{i,t} \times 100 = \alpha_{ind} + \alpha_t + \beta_1 ln\text{BTR}_{i,t-1} + \beta_2 ln\text{BTR}_{i,t-1} \times \text{BMT}_{t-1} + \gamma' \text{Controls}_{i,t-1} + \varepsilon_{i,t}. \quad \text{(E.2)}$$

As shown by Table E.3, we find that the coefficients for the interaction terms (β_2) are significantly negative, suggesting that the difference in CEO turnover rates between high BTR firms and low BTR firms is indeed larger conditional on worse aggregate funding liquidity condition. This interaction effect is economically significant. For example, according to the specification with industry and year fixed effects, when BMT returns changes from its mean value (-5.5%) to a value that is two standard deviation above the mean (26.5%), the sensitivity between lnBTR and CEO turnover (Turnover$^{(1)}$) more than triples (the coefficient changes from -0.729 to -2.747).

E.2 BTR and Managerial Turnovers

We also repeat the turnover analyses by including all managers in BoardEx.[3] The results are tabulated in Table E.4. Consistent with our previous analyses using CEOs and top 5 executives, we find that managerial turnovers are negatively associated with BTR. Moreover, this negative relation is more pronounced with worse aggregate funding liquidity conditions (i.e., when BMT returns are more positive).

[3]we exclude independent board members from our analyses

Table E.2: BTR and CEO turnovers

	(1)	(2)	(3)	(4)
	$\text{Turnover}_t^{(1)} \times 100$		$\text{Turnover}_t^{(2)} \times 100$	
$ln\text{BTR}_{t-1}$	-0.902^{***}	-0.870^{***}	-0.899^{***}	-0.881^{**}
	[−3.948]	[−3.240]	[−2.841]	[−2.595]
$ln(\text{OC/Asset})_{t-1}$	0.143	0.151	0.221	0.156
	[0.629]	[0.539]	[1.044]	[0.588]
$ln\text{size}_{t-1}$	−0.110	0.144	−0.034	0.235
	[−0.560]	[0.570]	[−0.167]	[0.823]
$ln\text{BEME}_{t-1}$	0.105	0.674	0.395	0.986^{*}
	[0.229]	[1.216]	[0.848]	[1.836]
$ln\text{lev}_{t-1}$	0.360	0.415	0.484	0.537
	[1.244]	[1.003]	[1.709]	[1.274]
StockRet_{t-1}	-4.258^{***}	-4.274^{***}	-4.217^{***}	-4.207^{***}
	[−3.505]	[−3.224]	[−3.654]	[−3.299]
Female	0.259	−0.534	0.521	−0.284
	[0.245]	[−0.480]	[0.434]	[−0.229]
Industry FE	No	Yes	No	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	4875	4875	4875	4875
R-squared	0.012	0.028	0.012	0.031

Note: This table shows the relation between BTR and CEO turnovers. We study the turnovers of CEOs covered by the Execucomp data. We match Execucomp with BoardEx and use the employment history data in BoardEx to identify executive turnovers. In Column (1) and (2), the dependent variables are 100 for a given CEO-year observation if the CEO leaves the firm at age 59 or younger due to reasons other than death, and it is 0 otherwise. In Column (3) and (4), the dependent variables are 100 for a given CEO-year observation if the CEO leaves the firm at age 59 or younger due to reasons other than death, or if the CEO resigns according to the Execucomp data, and it is 0 otherwise. The main independent variable is the lagged lnBTR. lnBTR is the natural log of the ratio between brand stature and brand strength. Brand stature and brand strength are two brand metrics constructed by the BAV Group based on its comprehensive brand perception survey. Brand stature measures brand loyalty of existing customers. Brand strength measures how much the brand is perceived by the consumers to be innovative and distinctive. We standardize lnBTR to ease the interpretation of the coefficients. Control variables include lagged firm characteristics such as the natural log of the organization-capital-to-asset ratio $ln(\text{OC/Asset})$, the natural log of firm market capitalization (lnsize), the natural log of the book-to-market ratio (lnBEME), the natural log of the debt-to-equity ratio (lnlev), the 12-month stock returns in the previous year (StockRet), and a dummy variable for the gender of the CEOs (Female). We include SIC-2 industry fixed effects and year fixed effects in the regressions. Our sample includes firms that are listed on NYSE, AMEX, and NASDAQ exchanges with share codes 10 or 11. We exclude financial firms and utility firms from the analysis. The sample period is 1993 to 2016. We include t-statistics in parentheses. Standard errors are clustered by firm and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

Table E.3: BTR and CEO turnovers: interaction with BMT

	(1)	(2)	(3)	(4)
	$\text{Turnover}_t^{(1)} \times 100$		$\text{Turnover}_t^{(2)} \times 100$	
$ln\text{BTR}_{t-1}$	-1.099^{***}	-1.076^{***}	-1.112^{***}	-1.088^{**}
	[−4.343]	[−3.486]	[−3.057]	[−2.622]
$ln\text{BTR}_{t-1} \times \text{BMT}_{t-1}$	-6.100^{***}	-6.305^{***}	-6.738^{***}	-7.031^{***}
	[−3.549]	[−3.656]	[−3.170]	[−3.283]
$ln(\text{OC/Asset})_{t-1}$	0.263	0.129	0.340	0.137
	[1.072]	[0.439]	[1.503]	[0.488]
$ln\text{size}_{t-1}$	−0.193	0.046	−0.116	0.141
	[−1.000]	[0.175]	[−0.566]	[0.487]
$ln\text{BEME}_{t-1}$	0.498	1.007	0.747	1.272^{**}
	[0.981]	[1.657]	[1.487]	[2.150]
$ln\text{lev}_{t-1}$	0.603	0.697	0.717^{*}	0.809^{*}
	[1.682]	[1.497]	[2.067]	[1.725]
StockRet_{t-1}	-4.355^{***}	-4.315^{***}	-4.345^{***}	-4.287^{***}
	[−2.979]	[−2.819]	[−3.028]	[−2.831]
Female	−0.027	−0.908	0.222	−0.677
	[−0.026]	[−0.811]	[0.187]	[−0.535]
Industry FE	No	Yes	No	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	4931	4931	4931	4931
R-squared	0.015	0.030	0.015	0.033

Note: This table shows the relation between CEO turnovers and the interaction between BTR and BMT. BMT is the yearly returns for the brand-minus-talent (Quintile 5 − Quintile 1 BTR) portfolio. The mean of BMT is −0.055 (i.e., −5.5%), while the standard deviation of BMT is 0.160 (i.e., 16.0%). We study the turnovers of CEOs covered by the Execucomp data. We match Execucomp with BoardEx and use the employment history data in BoardEx to identify executive turnovers. In Column (1) and (2), the dependent variables are 100 for a given CEO-year observation if the CEO leaves the firm at age 59 or younger due to reasons other than death, and it is 0 otherwise. In Column (3) and (4), the dependent variables are 100 for a given CEO-year observation if the CEO leaves the firm at age 59 or younger due to reasons other than death, or if the CEO resigns according to the Execucomp data, and it is 0 otherwise. The main independent variables are the lagged *ln*BTR, and the products between the lagged *ln*BTR and the lagged BMT. *ln*BTR is the natural log of the ratio between brand stature and brand strength. Brand stature and brand strength are two brand metrics constructed by the BAV Group based on its comprehensive brand perception survey. Brand stature measures brand loyalty of existing customers. Brand strength measures how much the brand is perceived by the consumers to be innovative and distinctive. We standardize *ln*BTR to ease the interpretation of the coefficients. Control variables include lagged firm characteristics such as the natural log of the organization-capital-to-asset ratio $ln(\text{OC/Asset})$, the natural log of firm market capitalization (*ln*size), the natural log of the book-to-market ratio (*ln*BEME), the natural log of the debt-to-equity ratio (*ln*lev), the 12-month stock returns in the previous year (StockRet), and a dummy variable for the gender of the CEOs (Female). We include SIC-2 industry fixed effects and year fixed effects in the regressions. Our sample includes firms that are listed on NYSE, AMEX, and NASDAQ exchanges with share codes 10 or 11. We exclude financial firms and utility firms from the analysis. The sample period is 1993 to 2016. We include t-statistics in parentheses. Standard errors are clustered by firm and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

Table E.4: BTR and managerial turnovers in the BoardEx data

	(1)	(2)	(3)	(4)
	Turnover$_t$ × 100			
lnBTR$_{t-1}$	-1.928^{***}	-1.324^{***}	-2.026^{***}	-1.374^{***}
	[−6.154]	[−3.378]	[−6.444]	[−3.481]
lnBTR$_{t-1}$ × BMT$_{t-1}$			-1.564^{**}	-0.829^{**}
			[−2.536]	[−2.218]
ln(OC/Asset)$_{t-1}$	0.421^{**}	0.228	0.416^{**}	0.229
	[2.685]	[1.123]	[2.661]	[1.101]
lnsize$_{t-1}$	0.939^{***}	0.589^{***}	0.945^{***}	0.597^{***}
	[5.815]	[3.156]	[5.860]	[3.176]
lnBEME$_{t-1}$	1.467^{***}	1.830^{***}	1.509^{***}	1.862^{***}
	[4.356]	[5.742]	[4.453]	[5.813]
lnlev$_{t-1}$	0.662	0.915^{**}	0.661	0.917^{**}
	[1.693]	[2.783]	[1.695]	[2.782]
StockRet$_{t-1}$	-1.705^{***}	-1.905^{***}	-1.669^{**}	-1.894^{***}
	[−2.895]	[−3.636]	[−2.797]	[−3.639]
Female	−0.053	0.092	−0.053	0.094
	[−0.252]	[0.424]	[−0.251]	[0.433]
Industry FE	No	Yes	No	Yes
Year FE	Yes	Yes	Yes	Yes
Observations	421269	421269	419195	419195
R-squared	0.009	0.013	0.009	0.013

Note: Column (1) and (2) of this table show the relation between managerial turnovers and BTR. Column (3) and (4) of this table show the relation between managerial turnovers and the interaction between BTR and BMT. We identify managerial turnovers using the employment history data in BoardEx. The dependent variables are 100 for a given manager-year observation if the manager leaves the firm at age 59 or younger due to reasons other than death, and it is 0 otherwise. BMT is the yearly returns for the brand-minus-talent (Quintile 5 − Quintile 1 BTR) portfolio. The mean of BMT is −0.055 (i.e., −5.5%), while the standard deviation of BMT is 0.160 (i.e., 16.0%). lnBTR is the natural log of the ratio between brand stature and brand strength. Brand stature and brand strength are two brand metrics constructed by the BAV Group based on its comprehensive brand perception survey. Brand stature measures brand loyalty of existing customers. Brand strength measures how much the brand is perceived by the consumers to be innovative and distinctive. We standardize lnBTR to ease the interpretation of the coefficients. Control variables include lagged firm characteristics such as the natural log of the organization-capital-to-asset ratio ln(OC/Asset), the natural log of firm market capitalization (lnsize), the natural log of the book-to-market ratio (lnBEME), the natural log of the debt-to-equity ratio (lnlev), the 12-month stock returns in the previous year (StockRet), and a dummy variable for the gender of the managers (Female). We include SIC-2 industry fixed effects and year fixed effects in the regressions. Our sample includes firms that are listed on NYSE, AMEX, and NASDAQ exchanges with share codes 10 or 11. We exclude financial firms and utility firms from the analysis. The sample period is 1993 to 2016. We include t-statistics in parentheses. Standard errors are clustered by firm and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

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