# Online Appendix for "Job Search under Debt: Aggregate Implications of Student Loans"

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## A Data

My empirical analysis uses panel data from the National Longitudinal Survey of Youth 1997 (NLSY97). This is a nationally representative survey conducted by the Bureau of Labor Statistics. In round 1, 8,984 youths were initially interviewed in 1997. Follow-up surveys were conducted annually. Almost 83% (7,423) of the round 1 sample were interviewed in round 15 (2011-2012). Youths were born between 1980 and 1984. Their ages ranged from 12 to 18 in round 1 and were 26 to 32 in round 15.

My analysis focuses on high school and college graduates. I do not include college dropouts because it is not clear when they enter the labor market. I drop youths who have ever served in the military or attended graduate schools because they are not in the same position as the other youths in my sample when it comes to making labor market decisions. I also drop youths who received the bachelor's degree before 1997 due to the lack of labor market information upon college graduation. This leaves me with a sample of 1,721 high school graduates and 1261 college graduates.

All dollar-denominated variables are converted to 2009 dollars using GDP deflator. The details of the data construction follow.

### A.1 Variables Used for Sample Selection

**Highest degree** In each year, NLSY97 collects the highest degree received to the start of the interview year. The cumulative variable CVC_HIGHEST_DEGREE_EVER documents the highest degree received ever according to the most recent survey. I only keep the youths with a bachelor's degree (CVC_HIGHEST_DEGREE_EVER=4) or a high school degree (CVC_HIGHEST_DEGREE_EVER=2).

**Military service** I check two variables for military services. The variable YCPS_2400, available in years 1997, 2000, 2006, documents whether the youth is now in the active Armed Forces. I drop the the youths who answered yes in any of these surveys. The variable YEMP_59000, available in years 1998-2012, documents whether the youth is/was in the regular, the Reserves, or the National Guard. I drop youths who ever had these statuses.

**Enrollment in grad schools** Some youths choose to continue a graduate program after college graduation. I drop these students because their labor market experience is likely to be different. The variable CV_ENROLLSTAT, available in each year since 1997, documents the enrollment status as of the survey year. I drop youths who ever enrolled in a graduate program (CV_ENROLLSTAT=11).

**Degree receiving date** The variable CVC_BA_DEGREE documents the date on which the youth received a bachelor' s degree in a continuous month scheme. I drop youths who received the bachelor's degree before 1997 due to the lack of labor market information upon college graduation.

## A.2 Variables Used for Model Estimation and Regression Analyses

### A.2.1 Variables Used for Both Model Estimation and Regression Analyses

**Student loan debt** I construct the student loan debt variable following Addo (2014). The variable YSCH_25600 documents the amount of loans borrowed in government-subsidized loans or other types of loans while the youth attended schools/institutions in each term and each college. Together with the records on enrollment information, I construct the amount of student loans taken out in each year and the total amount of student loans borrowed before college graduation. Unfortunately, there is no information on repayment in the data. Because students rarely repay student loan debt during college, I consider the total amount of student loans borrowed as the amount of outstanding student loan debt upon college graduation. To prevent the skewness of the debt distribution having a large effect on the estimated means, the total amount of student loan debt is top coded at 99 percentile ($49,280).

**Last date enrolled** I construct a "last-enrolled" variable to record the last date on which the youth is in school. I consider the youth as in the labor market after this date is passed. For college graduates, the variable SCH_COLLEGE_STATUS documents the youth's college enrollment status in each month since 1997. Based on this information, I set the value of "last-enrolled" to be the latest month that the youth was enrolled in college (SCH_COLLEGE_STATUS=3). Then, I check whether the value of "last-enrolled" variable is consistent with the date that the youth receives her bachelor's degree, documented by the variable CVC_BA_DEGREE. Among the 1261 college graduates in my sample, 83 youths have the last date enrolled being inconsistent with the degree date for at least 1 year. These youths are not considered when constructing the labor market moments below. For high school graduates, I use the high school degree receiving date as the last date in school.

**Duration of unemployment spells** I construct the duration of unemployment spells by tracking the period until an unemployed (or out of the labor force) youth finds a job. In my sample, there are 7,969 unemployment spells with an average duration of 27.2 weeks and a standard deviation of 59.4 weeks.

**Wage income** The variable YINC_1700 documents income that the youth received from wages, salary, commissions, or tips from all jobs in past year, before deductions for taxes or anything else. This is the variable I use to construct annual wage income. An alternative method to construct annual wage income is to use the information on hours and hourly wage rate. The two methods usually provide different numbers due to measurement errors. I prefer to use the variable YINC_1700 to construct annual wage income because the value of this variable is directly obtained from the questionnaire but the second method uses data constructed by BLS staff based on several discretionary assumptions. To be consistent, I construct an average hourly wage rate by dividing deflated values of YINC_1700 by the total number of hours worked in that year. When constructing annual wage income for each youth, I follow Rubinstein and Weiss (2006) by excluding the youths whose hourly wage rates are below $4 or higher than $2,000 and who worked less than 35 weeks or less than 1,000 annual hours.

**Hours** The variable EMP_HOURS documents the total number of hours worked by a youth at any job in each week. Hours per week worked at each job are assumed constant except during a reported gap, when the hours for that job are assumed to be zero. Weekly hours are top coded at 140 hours.

### A.2.2 Variables Used Only for Model Estimation

**Net liquid wealth** I construct the net liquid wealth variable using financial assets. Loans received from family members and friends to help pay for college are not subtracted in the measure of net liquid wealth. This is because, as argued by Johnson (2013), it is not clear whether or when these youths would need to repay the loans from family members and friends for educational purposes. I do not include non-financial assets, e.g., housing and property values, farm operation, etc., because these assets are not as liquid, and accounting for their values downplays the marginal propensity to consume. As I show in section 3, the repayment of student loans affects job search strategy through the liquidity channel, which depends on the marginal propensity to consume.

The variable CVC_ASSETS_FINANCIAL documents the value of financial assets when the youth reaches ages 18, 20, and 25. The financial assets include savings and checking accounts, money market funds, retirement accounts, stocks, bonds, and life insurance, etc. I use the value of financial assets at age 18 to proxy the net liquid wealth right before making the college entry decision. To prevent the skewness of the asset distribution having a large effect on the estimated means, the net liquid wealth values are top coded at 99 percentile ($69,695).

One concern is that money in retirement accounts is not as liquid. The adjustment is made using the variable YAST_4292, which documents the amount of savings in pension/retirement plans. Making this adjustment has almost no effect on the distribution of liquid wealth because only 50 youths reported to have positive balance in these plans with an average amount of $39.7.

**Work status** I construct the youth's work status using the variable EMP_STATUS, which documents the youth's weekly employment status since 1997. This variable documents whether the youth is employed, unemployed, or out of the labor force. Because my model does not distinguish between unemployed and out of the labor force, I consider the youths who are out of the labor force as unemployed. For employed youths, the associated employer number is also documented.

**Duration of employment spells** For each youth, I construct the duration of her employment spells by tracking the period between the date of moving from unemployment status to employment status and the date of moving from employment status to unemployment status. I drop employment spells whose duration is less than five weeks, because these are likely to be temporary or insecure jobs. In my sample, there are 8,130 employment spells with an average duration of 113.2 weeks and a standard deviation of 136.2 weeks.

**Job tenure** For each youth, I construct her tenure at each job (employer) by tracking the period between the date of moving to the job and the date of leaving the job. In my sample, there are 12,086 job spells with an average duration of 76.3 weeks and a standard deviation of 106.9 weeks.

**Hourly wage rate** The variable CV_HRLY_PAY documents the hourly rate of pay as of either the job's stop date or the interview date for on-going jobs. This variable is used to construct the wage increase upon job-to-job transitions (not wage income; see above).

**Wage increase upon job-to-job transitions** I construct the log wage increase upon job-to-job transitions by calculating the change in log hourly wage rate between consecutive job spells.

**Government benefits** The monthly take-up status and benefit amount of AFDC, food stamps, and WIC between 1997-2009 are documented in variables, AFDC_AMT, AFDC_STATUS, FDSTMPS_AMT, FDSTMPS_STATUS, WIC_AMT, WIC_STATUS.

**Others** The remaining moments are constructed using other data sources. The vacancy to unemployment ratio is constructed using job openings information since December 2000 from JOLTS. The life-cycle earnings profile between ages 23-60 is constructed using March CPS 1997-2008 from Acemoglu and Autor (2011) (available on David Autor's website).

### A.2.3 Variables Used Only for Regression Analyses

**Parental wealth and education** The variable CV_HH_NET_WORTH_P documents household net worth from parent interview in 1997. I use this variable to proxy parental wealth. The variable, CV_HGC_BIO_DAD and CV_HGC_BIO_MOM, document the highest grade completed by each youth's biological father and mother. I use the mean of the two variables to proxy parental education.

**Gender, race, age, and AFQT score** can be found from variables, KEY!SEX, KEY!RACE_ETHNICITY, KEY!BDATE, ASVAB_MATH_VERBAL_SCORE_PCT.

**County of residence** is available from NLSY restricted geocode CD. The variable GEO01 documents the youth's residence in each survey year.

**Job industry** The variable YEMP_INDCODE_2002 documents the 4-digit business or industry code based 2002 Census Industry Codes for each youth between 1997-2013. Industry codes between 6870-6990 are classified as finance and banking jobs and those between 7270-7460 are classified as consulting jobs.

**Length of college study** The length of college study is constructed by taking the difference between the first date enrolled in college, available from variable SCH_COLLEGE_STATUS, and the BA degree receiving date, documented by variable CVC_BA_DEGREE.

**Sector** The variable YEMP_58500 documents whether the worker is employed by government, a private company, a nonprofit organization , or is working without pay in a family business or farm since 1997. I consider the respondent as working in the public sector if she is employed by government or by a nonprofit organization. There is only one respondent working without pay in a family business or farm. This data point is dropped when running regressions.

**College major** Respondents in rounds 1-13 (1997-2009) indicated their college majors from a pick list. The variable YSCH_21300 documents the youth's major field in each college each term since the date of last interview. Beginning in round 14 (2014), respondents' majors were collected in a verbatim format and then coded using the CIP (Classification of Instructional Programs) 2010 codes under the variable YSCH_21300_COD. In my sample, only 7 youths received the BA degree after 2010 (the most recent graduate received his degree in September 2011). For these youths, I use the majors recorded before round 14 to be consistent with the old coding system. Among the rest 1254 youths, 1234 youths' majors are documented in at least one of the survey between 1997-2009. For the 104 youths who changed majors during college study, I use the most recently reported major before the degree receiving date to represent the major associated with the BA degree. The old coding system has a very fine category with 45 different majors, which generates a collinearity problem (with the county fixed effect) in my wage regressions because of the small sample size. Therefore, I reclassify the recorded majors into four broader category, including physical science, social science, engineering, and others.

### A.3 Adjusting the Higher-Order Moments for Unmodeled Variation

In the model, the exogenous sources of variation among agents come from differences in initial wealth, talent, student loan debt, and histories of shocks to job offers. By contrast, the data contain unmodeled variation due to heterogeneity in personal characteristics, family background, occupation, and industry fixed effects. Ignoring these sources of variation would not be problematic if the moments used in identification only include sample averages. However, because the talent and vacancies' productivity distribution are identified using the second (variation) and third (skewness) moments of the cross-sectional log wage income distribution and the variance of log wage increase upon job-to-job transitions, ignoring these sources of variation would bias the estimation result. Intuitively, failure to account for the unmodeled variation in the data would result in a more dispersed estimated productivity distribution, which will in turn exaggerate the option value of staying unemployment and overestimate the effect of the debt burden on job search decisions.

I adjust the data by purging the unmodeled sources of variation from the data following the approach of Gourinchas and Parker (2002) and Kaboski and Townsend (2011). In particular, I run linear regressions of log wage income. The estimated equation is:

$$\log Wage_{i,t} = \beta_w X_{i,t} + \epsilon_{w,i,t}, \tag{A.1}$$

where $X_{i,t}$ is a vector of controls including race, gender, parental net worth and education, occupation, and year fixed effects. I then construct the adjusted data for individuals with mean values of the explanatory variables ($\overline{X}$) using the estimated coefficients and residuals:

$$\widetilde{\log Wage}_{i,t} = \hat{\beta}_w \overline{X} + \hat{\epsilon}_{w,i,t}.$$

Finally, I construct the variance and skewness moments of the cross-sectional log wage income distribution using the adjusted log wage income $\widetilde{\log Wage}_{i,t}$.

### A.4 Suggestive Evidence

In this subsection, I present the full regression table for the validation test conducted in section 5.3.1.

Table A.1: The duration of the first unemployment spell after college graduation.

| | Duration of the first unemployment spell | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| Loan amount | -1.54** | -2.08*** | -1.92*** |
| (in $10,000) | (0.66) | (0.68) | (0.63) |
| Parental wealth | -0.02 | -0.00 | 0.03 |
| (in $10,000) | (0.06) | (0.07) | (0.08) |
| Parental education | 0.36 | 0.68 | 0.57 |
| | (0.41) | (0.53) | (0.53) |
| Female | | 3.37 | 1.91 |
| | | (2.23) | (2.27) |
| AFQT | | -0.01 | -0.03 |
| | | (0.06) | (0.06) |
| Race: Black | | -0.23 | -2.10 |
| | | (5.24) | (4.09) |
| Hispanic | | 2.62 | 2.92 |
| | | (9.49) | (9.18) |
| Mixed Race | | 1.56 | 3.51 |
| | | (4.00) | (3.60) |
| Married | | 1.00 | -0.81 |
| | | (3.41) | (3.29) |
| age | | -28 | -148 |
| | | (271) | (227) |
| $age^2$ | | 1.38 | 6.17 |
| | | (10.91) | (9.04) |
| $age^3$ | | -0.02 | -0.08 |
| | | (0.15) | (0.12) |
| Major: Physical Science | | | 6.55 |
| | | | (4.31) |
| Social Science | | | 4.35 |
| | | | (2.70) |
| Others | | | 5.71* |
| | | | (3.28) |
| Industry: finance, banking, | | | -6.78*** |
| and consulting | | | (2.02) |
| Length of college study | | | 0.42 |
| | | | (0.58) |
| Observations | 884 | 771 | 728 |
| County fixed effect | √ | √ | √ |
| $R^2$ | 0.0057 | 0.0183 | 0.0291 |

Note: This table examines the impact of student loan debt on the duration of the first unemployment spell after college graduation. A $10,000 increase in the amount of student loans reduces the duration of the first unemployment spell by about 2 weeks. Each observation is at the individual level. The dependent variable is the number of weeks elapsed from the college graduation date to the date of starting the first full-time job (i.e., work more than 35 hours per week for at least two consecutive weeks). The dependent variable is regressed on the total amount of student loan debt borrowed during college study, recorded in units of $10,000. All regressions control for parental wealth, parental education, and the county of residence in the graduation year. Column (2) adds additional controls for gender, race, AFQT score, marital status, and the cubic age polynomials. Column (3) adds additional controls for college major, job industry, and the length of college study. Standard errors are clustered at the county level. ***, **, and * indicate significance at the 1, 5, and 10 percent level.

Table A.2: The impact of student loan debt on post-graduation wage income.

| | First year | | | Second year | | | Third year | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (1) | (2) | (3) | (1) | (2) | (3) |
| Loan amount | -1,830** | -2,067** | -2274** | -1,812** | -2,152** | -2,232** | -2,009* | -2,619** | -2,821** |
| (in $10,000) | (770) | (890) | (920) | (789) | (865) | (882) | (1,117) | (1,309) | (1,372) |
| Parental wealth | 100* | 94* | 77 | 91 | 106 | 95 | 53 | 33 | 56 |
| (in $10,000) | (56) | (55) | (56) | (70) | (84) | (69) | (85) | (83) | (90) |
| Parental education | 19 | -376 | -146 | 290 | -364 | -130 | 611 | -29 | 320 |
| | (305) | (380) | (405) | (389) | (523) | (516) | (538) | (623) | (565) |
| Female | | -6,140*** | -3,585* | | -6,347*** | -3,135 | | -8,154*** | -4,738* |
| | | (1,969) | (1,864) | | (2,142) | (2,155) | | (2,765) | (2,513) |
| AFQT | | 80.7 | 55.4 | | 112.0 | 94 | | 117 | 108 |
| | | 52.6 | (51.8) | | (69.5) | (68) | | (78) | (74) |
| Race: Black | | 1,491 | 52 | | -835 | -142 | | 992 | 1,613 |
| | | (3,679) | (3,741) | | (4,986) | (4,825) | | (5,340) | (5,931) |
| Hispanic | | -730 | -696 | | -8,496 | -5,825 | | -12,583 | -6,366 |
| | | (8,473) | (8,049) | | (8,113) | (8,049) | | (13,008) | (11,574) |
| Mixed Race | | 2,051 | 513 | | -1,323 | -2,841 | | 1,326 | -446 |
| | | (2,850) | (2,820) | | (3,515) | (3,335) | | (4,102) | (4,129) |
| Married | | -1,153 | -2,415 | | -2,337 | -2,081 | | -4,563 | -4,860 |
| | | (2,457) | (2,469) | | (3,349) | (3,166) | | (3,616) | (3,871) |
| age | | -9.3e4 | 1.5e4 | | 2.4e5 | -2.3e5 | | 9.9e4 | 5.9e5 |
| | | (3.0e5) | (3.0e5) | | (4.2e5) | (4.8e5) | | 1.1e6 | 1.6e6 |
| $age^2$ | | 3.4e3 | -1.0e3 | | 1.0e4 | 9.8e3 | | -3.3e3 | -2.4e4 |
| | | (1.2e4) | (1.2e4) | | (1.8e4) | (2.0e4) | | (4.7e4) | 6.6e4 |
| $age^3$ | | -42 | 18 | | -145 | -138 | | 33 | 323 |
| | | (163) | (163) | | (244) | (276) | | (662) | 929 |
| Major: Physical Science | | | -20,189*** | | | -19,244*** | | | -20,969*** |
| | | | (4,988) | | | (4,631) | | | (6,697) |
| Social Science | | | -20,370*** | | | -21,147*** | | | -23,233*** |
| | | | (4,627) | | | (4,512) | | | (6,453) |
| Others | | | -24,729*** | | | -26,608*** | | | -28,201*** |
| | | | (4,532) | | | (5,184) | | | (6,708) |
| Industry: finance, banking, | | | 5,632*** | | | 5,498** | | | 4,358 |
| and consulting | | | (2,158) | | | (2,615) | | | (3,088) |
| Length of college study | | | 495 | | | -536 | | | -164 |
| | | | (563) | | | (647) | | | (863) |
| Observations | 671 | 596 | 582 | 588 | 518 | 507 | 483 | 427 | 415 |
| County fixed effect | √ | √ | √ | √ | √ | √ | √ | √ | √ |
| $R^2$ | 0.0175 | 0.0651 | 0.1455 | 0.0221 | 0.0733 | 0.1361 | 0.0185 | 0.0713 | 0.1311 |

Note: This table examines the impact of student loan debt on wage income in the first three years after college graduation. A $10,000 increase in the amount of student loans reduces the annual wage income by about $2,000. The dependent variable is wage income in the $t$-th year ($t = 1, 2, 3$) after college graduation. The dependent variable is regressed on the total amount of student loan debt borrowed during college study, recorded in units of $10,000. All regressions control for parental wealth, parental education, and the county of residence in the graduation year. Column (2) adds additional controls for gender, race, AFQT score, marital status, and the cubic age polynomials. Column (3) adds additional controls for college major, job industry, and the length of college study. Standard errors are clustered at the county level. ***, **, and * indicate significance at the 1, 5, and 10 percent level.

Table A.3: The impact of student loan debt on first jobs' industry, sector, and labor supply.

| | **High-paid industry** | | | **Private sector** | | | **Labor supply** | | |
|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (1) | (2) | (3) | (1) | (2) | (3) |
| Loan amount | -0.005 | -0.032 | -0.033 | 0.026 | 0.037 | 0.031 | 13.0 | 24.9 | 20.2 |
| (in $10,000) | (0.047 | (0.051) | (0.052) | (0.061) | (0.069) | (0.070) | (23.8) | (29.4) | (29.0) |
| Parental wealth | 0.005* | 0.005* | 0.005 | -0.002 | -0.002 | -0.002 | 1.55 | 1.00 | 1.25 |
| (in $10,000) | (0.003) | (0.003) | (0.003) | (0.004) | (0.004) | (0.004) | (1.69) | (1.76) | (1.79) |
| Parental education | -0.008 | -0.041* | -0.042* | 0.067** | 0.041 | 0.041 | -9.4 | -30.1** | -29.5** |
| | (0.019) | (0.022) | (0.022) | (0.029) | (0.034) | (0.034) | (12.5) | (13.2) | (13.7) |
| Female | | -0.28*** | -0.24** | | -0.38** | -0.35** | | -235*** | -219*** |
| | | (0.11) | (0.11) | | (0.16) | (0.17) | | (60) | (60) |
| AFQT | | 0.006** | 0.006** | | -0.001 | -0.001 | | 1.68 | 1.40 |
| | | (0.003) | (0.003) | | (0.003) | (0.003) | | (1.30) | (1.32) |
| Race: Black | | 0.015 | -0.005 | | 0.103 | 0.107 | | 3.8 | -16.0 |
| | | (0.231) | (0.231) | | (0.319) | (0.322) | | (159.2) | (154.5) |
| Hispanic | | -0.270 | -0.170 | | 0.277 | 0.419 | | 12.3 | -3.8 |
| | | (0.579) | (0.578) | | (0.711) | (0.702) | | (184.0) | (193.2) |
| Mixed Race | | 0.011 | -0.004 | | 0.288 | 0.298 | | 59.1 | 42.2 |
| | | (0.193) | (0.195) | | (0.244) | (0.245) | | (99.4) | (101.4) |
| Married | | 0.118 | 0.132 | | -0.372* | -0.374* | | -130.5 | -158.6* |
| | | (0.152) | (0.153) | | (0.224) | (0.227) | | (91.4) | (82.4) |
| age | | -32.9* | -33.9* | | -44.0*** | -44.0** | | 748 | 3,825 |
| | | (18.6) | (18.7) | | (17.0) | (17.2) | | (7,066) | (7,085) |
| $age^2$ | | 1.37* | 1.42* | | 1.76*** | 1.77** | | -35.8 | -162.1 |
| | | (0.76) | (0.77) | | (0.68) | (0.69) | | (286.3) | (287.3) |
| $age^3$ | | -0.019* | -0.020* | | -0.023*** | -0.024** | | 0.545 | 2.242 |
| | | (0.010) | (0.011) | | (0.009) | (0.009) | | (3.851) | (3.863) |
| Major: Physical Science | | | 0.148 | | | 0.030 | | | -222.3 |
| | | | (0.246) | | | (0.415) | | | (147.1) |
| Social Science | | | -0.040 | | | -0.019 | | | -242.9* |
| | | | (0.209) | | | (0.354) | | | (131.7) |
| Others | | | -0.306 | | | -0.278 | | | -167.1 |
| | | | (0.232) | | | (0.374) | | | (135.8) |
| Length of college study | | | -0.027 | | | 0.021 | | | 42.3 |
| | | | (0.021) | | | (0.035) | | | (33.5) |
| Observations | 884 | 775 | 773 | 365 | 319 | 317 | 812 | 705 | 705 |
| County fixed effect | | | | | | | √ | √ | √ |
| $R^2$ | 0.0037 | 0.0417 | 0.0506 | 0.0142 | 0.0638 | 0.0694 | 0.0029 | 0.0383 | 0.0521 |

Note: This table examines the impact of student loan debt on the industry and sector of first jobs and the number of working hours in the first year after college graduation. There is no significant finding on these margins. The first three columns estimate a Probit model using whether the respondent's first job is in finance, banking, and consulting industry as the dependent variable. The next three columns estimate a Probit model using whether the respondent's first job is in private sector as the dependent variable. The last three columns estimate an OLS regression using the number of workings hours in the first year after college graduation as the dependent variable. The treatment variable is the total amount of student loan debt borrowed during college study, recorded in units of $10,000. All regressions control for parental wealth, parental education, and the county of residence in the graduation year. Column (2) adds additional controls for gender, race, AFQT score, marital status, and the cubic age polynomials. Column (3) adds additional controls for college major, job industry, and the length of college study. Standard errors in the last three columns are clustered at the county level. ***, **, and * indicate significance at the 1, 5, and 10 percent level.

# B Proofs

## B.1 Proof of Proposition 1

*Proof.* Rearranging equation (3.3), the reservation wage is implicitly determined by

$$1 = \frac{\beta}{1-\beta} \int_{w^*_{FIX}}^{\overline{w}} \frac{u(w-s) - u(w^*_{FIX} - s)}{u(w^*_{FIX} - s) - u(\theta - s)} dF(w). \tag{B.1}$$

Consider increasing debt by $\Delta s$, and denote the reservation wage corresponding to $s + \Delta s$ as $\hat{w}^*_{FIX}$, thus according to (B.1),

$$1 = \frac{\beta}{1-\beta} \int_{\hat{w}^*_{FIX}}^{\overline{w}} \frac{u(w-s-\Delta s) - u(\hat{w}^*_{FIX} - s - \Delta s)}{u(\hat{w}^*_{FIX} - s - \Delta s) - u(\theta - s - \Delta s)} dF(w). \tag{B.2}$$

Define $u_2(x) = u(x - \Delta s)$, we can rewrite (B.2) as

$$1 = \frac{\beta}{1-\beta} \int_{\hat{w}^*_{FIX}}^{\overline{w}} \frac{u_2(w-s) - u_2(\hat{w}^*_{FIX} - s)}{u_2(\hat{w}^*_{FIX} - s) - u_2(\theta - s)} dF(w). \tag{B.3}$$

Let $r(x)$ and $r_2(x)$ be the local absolute risk aversion for $u(x)$ and $u_2(x)$. Thus

$$\begin{aligned} r(x) &> r_2(x) \quad \text{If } u(\cdot) \text{ has IARA;} \\ r(x) &= r_2(x) \quad \text{If } u(\cdot) \text{ has CARA;} \\ r(x) &< r_2(x) \quad \text{If } u(\cdot) \text{ has DARA.} \end{aligned} \tag{B.4}$$

Taking DARA as an example, note that $\theta - s < w^*_{FIX} - s < w - s$ for all $w \in (w^*_{FIX}, \overline{w}]$, thus according to Pratt (1964, Theorem 1),

$$\begin{aligned} 1 &= \frac{\beta}{1-\beta} \int_{w^*_{FIX}}^{\overline{w}} \frac{u(w-s) - u(w^*_{FIX} - s)}{u(w^*_{FIX} - s) - u(\theta - s)} dF(w) \\ &> \frac{\beta}{1-\beta} \int_{w^*_{FIX}}^{\overline{w}} \frac{u_2(w-s) - u_2(w^*_{FIX} - s)}{u_2(w^*_{FIX} - s) - u_2(\theta - s)} dF(w). \end{aligned} \tag{B.5}$$

Then (B.3) and (B.5) imply

$$\int_{\hat{w}^*_{FIX}}^{\overline{w}} \frac{u_2(w-s) - u_2(\hat{w}^*_{FIX} - s)}{u_2(\hat{w}^*_{FIX} - s) - u_2(\theta - s)} dF(w) > \int_{w^*_{FIX}}^{\overline{w}} \frac{u_2(w-s) - u_2(w^*_{FIX} - s)}{u_2(w^*_{FIX} - s) - u_2(\theta - s)} dF(w). \tag{B.6}$$

Because $\int_{w^*_{FIX}}^{\overline{w}} \frac{u_2(w-s) - u_2(w^*_{FIX} - s)}{u_2(w^*_{FIX} - s) - u_2(\theta - s)} dF(w)$ is decreasing in $w^*_{FIX}$, this implies $\hat{w}^*_{FIX} < w^*_{FIX}$.

Note that Danforth (1974) extends the result of Pratt (1964) to multi-dimensional lotteries. By applying Danforth (1974, Theorem 2), we can obtain a more general result, which indicates that higher debt reduces the agent's reservation wage even in a perfect credit market.

As an extension, if we assume that borrowers are protected from limited liability, i.e., they do not need to make repayment during unemployment, then equation (B.1) can be written as

$$\begin{aligned}1 &= \frac{\beta}{1-\beta}\int_{w^*}^{\overline{w}} \frac{u(w-s)-u(w^*-s)}{u(w^*-s)-u(\theta)} dF(w) \\ &= \frac{\beta}{1-\beta}\int_{w^*}^{\overline{w}} \left[\frac{u(w-s)-u(\theta)}{u(w^*-s)-u(\theta)} - 1\right] dF(w).\end{aligned} \tag{B.7}$$

Equation (B.7) implies that an increase in $s$ increases the reservation wage $w^*$. This is the risk-shifting effect of debt proposed by Donaldson, Piacentino and Thakor (2016) (a related discussion is in footnote 7).

□

## B.2 Proof of Proposition 2

*Proof.* The mileage that CRRA utility buys me is that it is a homogeneous utility function with multiplicative scaling behavior. With CRRA utility, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, equation (3.5) becomes

$$(w^*_{IBR})^{1-\gamma} = \theta^{1-\gamma} + \frac{\beta}{1-\beta}\int_{w^*_{IBR}}^{\overline{w}} [w^{1-\gamma} - (w^*_{IBR})^{1-\gamma}] dF(w). \tag{B.8}$$

Clearly, $w^*_{IBR}$ does not depend on $\alpha$. Therefore, under the IBR, when the utility has CRRA, the agent's reservation wage is equal to the reservation wage of the agent who has no debt. This suggests that

$$w^*_{IBR} = w^*|_{s=0} > w^*_{FIX}, \tag{B.9}$$

where the last inequality is from Proposition 1 because CRRA utility has decreasing absolute risk aversion. Note that another way to see that the reservation wage does not depend on $\alpha$ when utility has CRRA is to calculate the absolute risk aversion for utility $u((1-\alpha)x)$, which is $\gamma/x$, not a function of $\alpha$. Then, according to the proof of Proposition 1, the reservation wage stays the same because the local absolute risk aversion does not change for any $x$ when $\alpha$ changes.

In fact, we can further show that the disposable reservation wage also satisfies $\overline{w}^*_{IBR} > \overline{w}^*_{FIX}$. This indicates that the liquidity channel plus the risk channel strictly dominates the optionality channel according to Proposition 3. This result is obtained by applying the following lemma to equation (3.7).

**Lemma 1.** *The reservation wage under IBR satisfies:*

$$w^*_{IBR} < \frac{s}{\alpha}, \tag{B.10}$$

*where $\alpha$ solves equation (3.4).*

*Proof.* According to equation (3.5), $w^*_{IBR}$ is determined by

$$u((1-\alpha)w^*_{IBR}) = \frac{(1-\beta)u((1-\alpha)\theta) + \beta\int_{w^*_{IBR}}^{\overline{w}} u((1-\alpha)w)dF(w)}{1-\beta F(w^*_{IBR})}. \tag{B.11}$$

Thus

$$(1-\alpha)w^*_{IBR} = u^{-1}\left[\frac{(1-\beta)u((1-\alpha)\theta) + \beta\int_{w^*_{IBR}}^{\overline{w}} u((1-\alpha)w)dF(w)}{1-\beta F(w^*_{IBR})}\right]. \tag{B.12}$$

Notice that

$$\frac{(1-\beta) + \beta\int_{w^*_{IBR}}^{\overline{w}} dF(w)}{1-\beta F(w^*_{IBR})} = 1, \tag{B.13}$$

thus, we can think of the LHS of equation (B.13) as probability weights, which are imposed on $u((1-\alpha)x)$ to generate the RHS of equation (B.12).

By Jensen's inequality, equation (B.12) can be written as

$$w^*_{IBR} < \frac{(1-\beta)\theta + \beta\int_{w^*_{IBR}}^{\overline{w}} wdF(w)}{1-\beta F(w^*_{IBR})}. \tag{B.14}$$

According to equation (D.17) and (3.4), $\alpha$ is determined by

$$\frac{s}{\alpha} = \frac{(1-\beta)\theta F(w^*_{IBR}) + \int_{w^*_{IBR}}^{\overline{w}} wdF(w)}{1-\beta F(w^*_{IBR})}. \tag{B.15}$$

Therefore,

$$\begin{aligned}\frac{s}{\alpha} - w^*_{IBR} &> \frac{(1-\beta)\int_{w^*_{IBR}}^{\overline{w}} wdF(w) - (1-\beta)\theta[1-F(w^*_{IBR})]}{1-\beta F(w^*_{IBR})}\\ &> \frac{(1-\beta)[1-F(w^*_{IBR})](w^*_{IBR}-\theta)}{1-\beta F(w^*_{IBR})}\\ &> 0.\end{aligned} \tag{B.16}$$

□

Using Lemma 1, the disposable reservation wage satisfies

$$\begin{aligned}\overline{w}^*_{IBR} - \overline{w}^*_{FIX} &= (1-\alpha)w^*_{IBR} - w^*_{FIX} + s\\ &> (1-\alpha)w^*_{IBR} - w^*_{IBR} + s\\ &= \alpha\left(\frac{s}{\alpha} - w^*_{IBR}\right)\\ &> 0.\end{aligned} \tag{B.17}$$

□

### B.3 Proof of Lemma 1

*Proof.* Proposition D.2 indicates that $I(x)$ is increasing in $x$ when $x < \hat{w}$ (note: $\hat{w}$ is the reservation wage chosen by a risk-neutral agent). Therefore, equation (3.4) implies

$$\alpha = \frac{S}{\beta I(w^*_{IBR})} < \frac{S}{\beta I(\theta)} = \frac{s}{\int_\theta^{\overline{w}} w dF(w)}. \tag{B.18}$$

The expected disposable wage offer under the two contracts are

$$E_{IBR} = \int_{(1-\alpha)\theta}^{(1-\alpha)\overline{w}} w dF_{IBR}(w) = \int_\theta^{\overline{w}} (1-\alpha) w dF(w); \tag{B.19}$$

$$E_{FIX} = \int_{\theta-s}^{\overline{w}-s} w dF_{FIX}(w) = \int_\theta^{\overline{w}} (w-s) dF(w). \tag{B.20}$$

Taking the difference,

$$E_{IBR} - E_{FIX} = s - \int_\theta^{\overline{w}} \alpha w dF(w) > 0, \tag{B.21}$$

according to equation (B.18). Moreover, because $s/\alpha$ is the unique solution to $F_{IBR}(w) = F_{FIX}(w)$ and $(1-\alpha)\theta > \theta - s$, $F_{IBR}(w)$ single crosses $F_{FIX}(w)$ from below, i.e.,

$$\begin{aligned} F_{IBR}(w) < F_{FIX}(w) \quad & \text{for } w < s/\alpha \\ F_{IBR}(w) > F_{FIX}(w) \quad & \text{for } w > s/\alpha. \end{aligned} \tag{B.22}$$

For $z \in [0, s/\alpha]$, the single-crossing property implies

$$\int_0^z F_{IBR}(w) dw < \int_0^z F_{FIX}(w) dw. \tag{B.23}$$

For $z \in (s/\alpha, \overline{w}]$,

$$\begin{aligned} 0 &< E_{IBR} - E_{FIX} \\ &= \int_0^\infty [1 - F_{IBR}(w)] dw - \int_0^\infty [1 - F_{FIX}(w)] dw \\ &= \int_0^z [1 - F_{IBR}(w)] dw - \int_0^z [1 - F_{FIX}(w)] dw + \int_z^\infty [1 - F_{IBR}(w)] dw - \int_z^\infty [1 - F_{FIX}(w)] dw \\ &< \int_0^z [1 - F_{IBR}(w)] dw - \int_0^z [1 - F_{FIX}(w)] dw \\ &= \int_0^z F_{FIX}(w) dw - \int_0^z F_{IBR}(w) dw. \end{aligned} \tag{B.24}$$

Note that the second inequality uses the single-crossing property, and the second equality uses an expectation formula derived below. For a continuous random variable $x$ taking only non-negative values,

$$
\begin{aligned}
E(x) &= \int_0^\infty x f(x) dx \\
&= \int_0^\infty (-x) d(1-F(x)) \\
&= [-x(1-F(x))]_0^\infty + \int_0^\infty [1-F(x)] dx.
\end{aligned}
\tag{B.25}
$$

The first term in bracket vanishes because

$$
1 - F(x) = o(\frac{1}{x}) \quad \text{as } x \to \infty. \tag{B.26}
$$

□

### B.4 Proof of Proposition 3

*Proof.* Consider the fixed repayment contract. The disposable reservation wage, $\overline{w}^*_{FIX}$, is determined by

$$
\begin{aligned}
u(\tilde{w}^*_{FIX}) &= u(\theta - s) + \frac{\beta}{1-\beta} \int_{\tilde{w}^*_{FIX}}^{w-s} [u(w) - u(\tilde{w}^*_{FIX})] dF_{FIX}(w) \\
&= u(\theta - s) + \frac{\beta}{1-\beta} \int_{\tilde{w}^*_{FIX}}^{\infty} [u(w) - u(\tilde{w}^*_{FIX})] dF_{FIX}(w) \\
&= u(\theta - s) + \frac{\beta}{1-\beta} \left[ M - u(\tilde{w}^*_{FIX}) - \int_{\tilde{w}^*_{FIX}}^{\infty} u'(w) d \left( \int_0^w F_{FIX}(x) dx \right) \right] \\
&= u(\theta - s) + \frac{\beta}{1-\beta} \left[ M - u(\tilde{w}^*_{FIX}) - \lim_{x \to \infty} u'(x) \int_0^x F_{FIX}(w) dw + u'(\tilde{w}^*_{FIX}) \int_0^{\tilde{w}^*_{FIX}} F_{FIX}(w) dw \right. \\
&\quad \left. + \int_{\tilde{w}^*_{FIX}}^{\infty} \left( \int_0^w F_{FIX}(x) dx \right) u''(w) dw \right],
\end{aligned}
\tag{B.27}
$$

where $M = \lim_{x \to \infty} u(x)$. The last two equalities are derived by doing integration by parts.

Rearranging the above equation,

$$
\begin{aligned}
u(\tilde{w}^*_{FIX}) = (1-\beta) u(\theta - s) + \beta & \left[ M - \lim_{x \to \infty} u'(x) \int_0^x F_{FIX}(w) dw + u'(\tilde{w}^*_{FIX}) \int_0^{\tilde{w}^*_{FIX}} F_{FIX}(w) dw \right. \\
& \left. + \int_{\tilde{w}^*_{FIX}}^{\infty} \left( \int_0^w F_{FIX}(x) dx \right) u''(w) dw \right].
\end{aligned}
\tag{B.28}
$$

Similarly, for IBR, we have

$$
\begin{aligned}
u(\tilde{w}^*_{IBR}) = (1-\beta) u((1-\alpha)\theta) + \beta & \left[ M - \lim_{x \to \infty} u'(x) \int_0^x F_{IBR}(w) dw + u'(\tilde{w}^*_{IBR}) \int_0^{\tilde{w}^*_{IBR}} F_{IBR}(w) dw \right. \\
& \left. + \int_{\tilde{w}^*_{IBR}}^{\infty} \left( \int_0^w F_{IBR}(x) dx \right) u''(w) dw \right].
\end{aligned}
\tag{B.29}
$$

Taking the difference between (B.28) and (B.29):

$$u(\tilde{w}^*_{IBR}) - u(\tilde{w}^*_{FIX}) = (1-\beta)\left[u((1-\alpha)\theta) - u(\theta - s)\right] - \beta \lim_{x\to\infty} u'(x) \int_0^x \left[F_{IBR}(w) - F_{FIX}(w)\right] dw$$
$$+ \beta \left[\int_{\tilde{w}^*_{IBR}}^{\infty} \left(\int_0^w F_{IBR}(x)dx\right) u''(w)dw - \int_{\tilde{w}^*_{FIX}}^{\infty} \left(\int_0^w F_{FIX}(x)dx\right) u''(w)dw\right].$$
$$+ \beta \left[u'(\tilde{w}^*_{IBR}) \int_0^{\tilde{w}^*_{IBR}} F_{IBR}(w)dw - u'(\tilde{w}^*_{FIX}) \int_0^{\tilde{w}^*_{FIX}} F_{FIX}(w)dw\right] \tag{B.30}$$

Because $\lim_{x\to\infty} u'(x) = 0$ and $\lim_{x\to\infty} \int_0^x \left[F_{FIX}(w) - F_{IBR}(w)\right] dw$ is finite,

$$\lim_{x\to\infty} u'(x) \int_0^x \left[F_{FIX}(w) - F_{IBR}(w)\right] dw = 0. \tag{B.31}$$

Thus,

$$u(\tilde{w}^*_{IBR}) - u(\tilde{w}^*_{FIX}) = (1-\beta)\left[u((1-\alpha)\theta) - u(\theta - s)\right]$$
$$+ \beta \left[\int_{\tilde{w}^*_{IBR}}^{\infty} \left(\int_0^w F_{IBR}(x)dx\right) u''(w)dw - \int_{\tilde{w}^*_{FIX}}^{\infty} \left(\int_0^w F_{FIX}(x)dx\right) u''(w)dw\right]$$
$$+ \beta \left[u'(\tilde{w}^*_{IBR}) \int_0^{\tilde{w}^*_{IBR}} F_{IBR}(w)dw - u'(\tilde{w}^*_{FIX}) \int_0^{\tilde{w}^*_{FIX}} F_{FIX}(w)dw\right]. \tag{B.32}$$

In equation (B.32), increasing $\tilde{w}^*_{IBR}$ increases the LHS by $u'(\tilde{w}^*_{IBR})$, more than the increase in the RHS, $\beta F_{IBR}(\tilde{w}^*_{IBR}) u'(\tilde{w}^*_{IBR})$. Thus, given $\tilde{w}^*_{FIX}$, there is a unique $\tilde{w}^*_{IBR}$, and whether it is greater or less than $\tilde{w}^*_{FIX}$ depends on the sign of the RHS conditional on $\tilde{w}^*_{FIX} = \tilde{w}^*_{IBR}$.

The first term is positive because $(1-\alpha)\theta > \theta - s$ according to Lemma 1. When $\tilde{w}^*_{FIX} = \tilde{w}^*_{IBR}$, the second term is

$$\beta \left[\int_{\tilde{w}^*_{FIX}}^{\infty} \left(\int_0^w F_{IBR}(x)dx - \int_0^w F_{FIX}(x)dx\right) u''(w)dw\right], \tag{B.33}$$

which is positive because $u''(w) < 0$ and $\int_0^w F_{IBR}(x)dx - \int_0^w F_{FIX}(x)dx < 0$ for all $w > \tilde{w}^*_{FIX}$ according to Lemma 1.

When $\tilde{w}^*_{FIX} = \tilde{w}^*_{IBR}$, the third term is

$$\beta u'(\tilde{w}^*_{FIX}) \left[\int_0^{\tilde{w}^*_{FIX}} F_{IBR}(w)dw - \int_0^{\tilde{w}^*_{FIX}} F_{FIX}(w)dw\right], \tag{B.34}$$

which is negative according to Lemma 1.

□

# C Estimation and Numerical Methods

In this appendix section, I discuss the estimation and numerical method for the quantitative model in section 4. Different from existing search-theoretic models, the quantitative model is developed to allow most of the parameters being estimated in partial equilibrium without iterating on the equilibrium job contact rates. This largely simplifies the computation and makes the estimation of the full general equilibrium model tractable. Below I first discuss the estimation method and its limitations. Then I discuss the numerical algorithm that solves the model.

## C.1 Estimation Method

The standard way to estimate an equilibrium search model is to iterate on the set of parameters $\Xi$ in order to minimize the objective function (5.5). However, this method is not sufficiently tractable due to the large number of parameters in $\Xi$ and the model complexity. The computation burden mainly comes from numerically searching for the equilibrium job contact rates, which are endogenously determined by the firms' job posting decisions and the workers' search decisions. Although searching for the equilibrium objects is not difficult in a standard search model, it is enormously time consuming in my model due to the many features introduced. If there are ways to estimate a subset of parameters without searching for the equilibrium, then the total estimation time would be possibly reduced. This is the logic that underlies my estimation method.

In particular, I estimate the model in two steps: first, I treat the equilibrium job contact rates $\lambda_u$ and $\lambda_e$ as parameters and estimate a subset of parameters

$$\Xi_p = \{A_0, A_1, \kappa, \xi, f_1^a, f_2^a, f_1^\rho, f_2^\rho, \phi, \eta, \mu_e, \sigma_e, \vartheta, \mu_k, \sigma_k, \mu_0, \mu_1, \mu_2, \mu_3\}$$

along with $\lambda_u$ and $\lambda_e$ to match the moments in Table 2 except for the vacancy to unemployment ratio. Second, I fix the values of $\Xi_p$ and estimate the rest parameters $\Xi_q = \Xi / \Xi_p = \{h_e, h_u, \chi, \nu\}$. I normalize $s_e$ to be 1, and the other three parameters are estimated to match the vacancy to unemployment ratio and the job contact rates $\lambda_u$ and $\lambda_e$, which are estimated in the first step. This is straightforward, because equation (4.11) indicates that $h_u = \frac{\lambda_u}{\lambda_e}$. Therefore, the second step only needs to estimate two parameters $\chi$ and $\nu$ to match two moments, $\lambda_u$ and the vacancy to unemployment ratio.

Essentially, in the first step, I estimate a partial equilibrium search model with exogenous job contact rates. In the second step, I estimate a general equilibrium search model with only two parameters. This estimation method is much faster because most parameters are estimated in the first step without searching for the equilibrium objects when parameters are optimized. This is because the only equilibrium objects are job contact rates, which are treated as parameters. The estimation in the second step needs to search for the equilibrium objects, but it is much easier now because only two parameters are left to be optimized.

### C.1.1 Discussions and Limitations

This two-step estimation method obtains the same result as the standard way of estimating all the parameters together because my quantitative model satisfies three conditions: first, the only equilibrium objects are job contact rates, which are estimated in the first step. Second, all the parameters estimated in the second step affect the model outcomes only through their impacts on job contact rates. Third, all the moments used in the second-step estimation can be exactly matched.

The first condition is satisfied because I assume that the productivity of vacancies is randomly drawn from an exogenous distribution $F(\rho)$. This ensures that the equilibrium vacancy distribution $V(\rho)$ is the same as $F(\rho)$. This condition would be violated if the productivity is not randomly drawn. For example, if different firms can post vacancies of different productivity as in Lise and Robin (2017), then the vacancy distribution is also endogenous. As a result, there is no way to execute the first step due to the unknown vacancy distribution.[1] The limitation of assuming that vacancies' productivity is randomly drawn is that the model cannot capture the potential change in the distribution of vacancies' productivity when repayment policy changes. That is, my model does not capture the possibility that firms would create more productive jobs because IBR motivates borrowers to search for these jobs, the general equilibrium effect proposed by Acemoglu and Shimer (1999, 2000).

The second condition is satisfied because the search intensity parameters and the vacancy posting cost do not affect either agents' or firms' decisions once the job contact rates are given. It is straightforward to prove that the third condition is also satisfied.[2] If the third condition is not satisfied, this two-step estimation is guaranteed to be inconsistent with the standard way of estimating all parameters together. This is because if we cannot adjust the parameters in the second step to perfectly match the job contact rates, it means that we are over-fitting the model in the first step by selecting those contact rates that could never be achieved in equilibrium. Moreover, if we cannot perfectly match the vacancy to unemployment ratio in the second step, then the estimation result could also be different depending on the weighting matrix. This is because when all the parameters are estimated together, we may want to sacrifice the matched moments in the first step in order to better match the moment in the second step, namely, the vacancy to unemployment ratio.

### C.1.2 Estimating Standard Errors

Once the two-step estimation is finished, standard errors of parameters can be constructed in the standard way.

First, I estimate the variance-covariance matrix $\widehat{COV}$ for all moments. Because the vector of moments in the data can be computed without knowing parameter values, $\widehat{COV}$ can be computed by bootstrapping the data directly without doing iterated MSM. Specifically, I calculate the moments $N = 200$ times by bootstrapping, then use these $N$ observations of moments to construct the variance-covariance matrix. There are two issues in estimating $\widehat{COV}$. First, moments are constructed using different data sources. The life-cycle moments are constructed using March CPS, the vacancy to unemployment ratio is constructed using JOLTS, the default rate is constructed using NSLDS, and the remaining moments are constructed using NLSY97. The covariance between moments constructed in different data sources is set to be zero. Second, the moments in NLSY97 are constructed using different number of observations due to missing

[1]This could be solved if in the first step we treat both the job contact rates and the vacancy distribution as parameters. But then the third condition would be violated because it is almost impossible to fit exactly the distribution of vacancy by selecting the vacancy posting cost.

[2]To see this, note that given $\lambda^u$ and $\lambda^e$, the equilibrium distributions $\phi^u(\Omega)$ and $\phi^e(\Omega, \rho)$ are unique in the stationary equilibrium. The unemployment rate $\overline{u}$ is determined by equation (4.30). Substituting equations (4.10-4.11) into equation (4.29), then $N_v$ is uniquely determined as a function of $\nu$, $\lambda^u$, $\lambda^e$, and the equilibrium distributions. Thus, there is a unique $\nu$ to match the vacancy to unemployment ratio. Because the number of matches $M$ is a function of $N_v$ and $\chi$ in equation (4.9), given $N_v$, $\chi$ is uniquely solved to match the job contact rates.

values. The covariance between any pair of moments is constructed by bootstrapping non-missing-value observations for both moments. Thus the assumption here is that values are missing randomly, though it is not likely to be true in reality.

In my estimation, I use a diagonal weighting matrix, $\hat{\Theta} = [\text{diag}(\widehat{COV})]^{-1}$, because covariance is not precisely estimated and may bias the estimated parameter values. The asymptotic variance-covariance matrix for MSM estimators $\hat{\Xi}_2$ is given by:

$$Q(\hat{\Theta}) = (\nabla^T \hat{\Theta} \nabla)^{-1} \nabla^T \hat{\Theta} \widehat{COV} \hat{\Theta}^T \nabla (\nabla^T \hat{\Theta}^T \nabla)^{-1}, \tag{C.1}$$

where $\nabla = \frac{\partial \hat{m}_S(\Xi_2)}{\partial \Xi_2}|_{\Xi_2 = \hat{\Xi}_2}$ is the Jacobian matrix of the simulated moments evaluated at the estimated parameters.[3] The first derivatives are calculated numerically by varying each parameter's value by 1%. The standard errors of $\hat{\Xi}_2$ are given by the square root of the diagonal elements of $Q(\hat{\Theta})$.

## C.2 Numerical Method

I solve the model numerically. The computational complexity of this model is extremely large because this is an equilibrium model with five state variables (wealth, debt, talent, job productivity, and the negotiation benchmark's productivity).[4]

In addition to the complexity introduced by five state variables, the model is hard to solve due to the violation of the linear sharing rule in the Nash bargaining problem. Therefore, for each possible worker-job combination, the algorithm needs to solve a maximization problem whose objective function does not have an analytical solution and is determined endogenously. In the following, I first present the numerical algorithm. Then I describe the initialization of value functions in the final period. Finally, I discuss the implementation of this algorithm.

### C.2.1 Algorithm

The model is solved by backward induction using the following algorithm:

**(1).** Guess the equilibrium job contact rates $\lambda_u$ for unemployed workers, and $\lambda_e = \frac{s^e}{s^u}\lambda_u$ for employed workers.

**(2).** Solve the value functions $U(\Omega)$, $W(\Omega, \rho, \rho')$, and $J(\Omega, \rho, \rho')$ in the following steps:

- **(2.1).** Guess wage functions $w(\Omega, \rho, \rho')$ for all $\Omega$, $\rho$, and $\rho'$.
- **(2.2).** Solve problems (4.25-4.27) by backward induction from $t = T$ to $t = 1$ to obtain $U(\Omega)$, $W(\Omega, \rho, \rho')$, $J(\Omega, \rho, \rho')$, and the corresponding policy functions.

[3] In general, the formula should also incorporate simulation errors, thus the variance-covariance matrix for MSM estimators also depends on the number of simulated agents (Gourieroux and Monfort, 1997). The formula I use does not consider simulation errors because instead of simulating a number of agents, I adopt the histogram method by simulating the distribution of characteristics. Therefore, as long as I focus on the stationary equilibrium, the simulation outcomes are not dependent on randomly drawn shocks.

[4] Loosely speaking, solving the model is as difficult as solving the quantitative models of Krusell, Mukoyama and Sahin (2010) and Lise and Robin (2017). Krusell, Mukoyama and Sahin (2010) do not model on-the-job search and Lise and Robin (2017) consider risk-neutral agents. But Krusell, Mukoyama and Sahin (2010) and Lise and Robin (2017) also consider aggregate shocks in their models, which I do not have.

**(2.3).** Solve the Nash bargaining problems (4.15-4.17) to obtain wage $w'(\Omega, \rho, \rho')$.

**(2.4).** If $w'(\Omega, \rho, \rho') = w(\Omega, \rho, \rho')$ for all $\Omega$, $\rho$, and $\rho'$, go to step (3); otherwise, go to step (2.1).

**(3).** Given initial distributions $\mho(a, b_0)$ and the computed value functions, solve the optimal college entry decisions. Then given the policy functions, forward simulate the model from $t = 1$ to $t = T$ to obtain distributions $\phi^u(\Omega)$ and $\phi^e(\Omega, \rho)$.

**(4).** Compute the equilibrium unemployment rate $\overline{u}$ using equation (4.30) and the aggregate level of search intensity $S$ using equation (4.8). Compute the probability of contacting a worker $q$ using the free entry condition (4.29).

**(5).** Substituting $S$ and $q$ into equations (4.9-4.11) to obtain the number of meetings $M$, the number of vacancies $N_v$, and the equilibrium job contact rates $\lambda_u'$.

**(6).** Check if $\lambda_u' = \lambda_u$. If not, go to step (1).

Because I focus on the stationary equilibrium, the value functions and policy functions across different generations are identical. The final period represents age T. When the model is solved in partial equilibrium, the job contact rates $\lambda_u$ and $\lambda_e$ are given as parameters. Thus only steps (2) and (3) are executed.

### C.2.2 Initialization of Value Functions

The value functions at age $T$ are initialized by assuming that the agent consumes all wealth in the end. In the simulation, all agents should have paid off the outstanding debt before reaching age $T$. To have a well-defined problem, I also need to specify what happens off the equilibrium, i.e., if there is outstanding debt left at age $T$. I assume that the agent needs to pay off all the outstanding debt if wealth at age $T$ is sufficient to make the payment. If wealth is not sufficient, I punish the agent to keep the level of consumption at the floor value $\underline{c}$, and the rest wealth is used to repay the debt.

Formally, the value for unemployed workers at age $T$ is:

$$U(\Omega) = \begin{cases} \dfrac{[(1+r)b + \varkappa\theta^{1-\tau} - (1+r_s)s]^{1-\gamma}}{1-\gamma} & \text{if } (1+r)b + \varkappa\theta^{1-\tau} \geq (1+r_s)s + \underline{c} \\ \dfrac{\underline{c}^{1-\gamma}}{1-\gamma} & \text{otherwise} \end{cases} \tag{C.2}$$

The agent dies after age $T$, and the worker-job match separates as a consequence. Therefore, the value of a filled job at age $T$ is

$$J(\Omega, \rho, \rho') = [z\rho - w(\Omega, \rho, \rho')]l, \tag{C.3}$$

where $l = \left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1}{\sigma+\tau}} w(\Omega, \rho, \rho')^{\frac{1-\tau}{\sigma+\tau}}$.

To calculate the value for employed workers at age $T$, I need to solve the Nash bargaining problem to obtain the wage functions at age $T$. This can be solved directly from a root-finding problem.[5] Depending on whether the agent's wealth is sufficient to repay the debt, there are two cases:

[5]Note that the wage functions at the final period $T$ can be solved directly from a root-finding problem because the agent

**Case 1: Insolvency** If the agent is employed at job $\rho$, the highest wage rate that job $\rho$ can offer is its marginal product of labor $z\rho$. If the agent could not repay the debt when being offered this wage rate, then she is insolvent and would consume $\underline{c}$. Note that because the current job's productivity is always higher than the negotiation benchmark's productivity, the agent would also be insolvent and consume $\underline{c}$ at the negotiation benchmark. In this scenario, the agent is indifferent between being employed at the current job $\rho$ or at the negotiation benchmark, thus the match surplus for the agent is zero. Moreover, the agent has no incentive to supply labor as this only increases repayment but not consumption. Thus the firm would also obtain zero match surplus. This implies that the value for employed workers at age $T$ is $\underline{c}^{1-\gamma}/(1-\gamma)$ in the case of insolvency.

To pin down the condition for insolvency, consider the highest wage rate $z\rho$ being offered by job $\rho$ to agent $\Omega$. The after-tax income is $\varkappa(z\rho)^{1-\tau}$. Substituting the optimal labor supply, $l = \left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1}{\sigma+\tau}} (z\rho)^{\frac{1-\tau}{\sigma+\tau}}$, the maximum wealth that the agent can obtain is

$$\begin{aligned}\bar{b} &= (1+r)b + \varkappa(z\rho l)^{1-\tau} \\ &= (1+r)b + \varkappa\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1-\tau}{\sigma+\tau}} (z\rho)^{\frac{(1+\sigma)(1-\tau)}{\sigma+\tau}}.\end{aligned} \tag{C.4}$$

The agent's utility is

$$\begin{aligned}u(\bar{b}-(1+r_s)s, l) &= \frac{1}{1-\gamma}\left[\bar{b}-(1+r_s)s-\phi\frac{l^{1+\sigma}}{1+\sigma}\right]^{1-\gamma} \\ &= \frac{1}{1-\gamma}\left[\bar{b}-(1+r_s)s-\frac{\phi}{1+\sigma}\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1+\sigma}{\sigma+\tau}} (z\rho)^{\frac{(1-\tau)(1+\sigma)}{\sigma+\tau}}\right]^{1-\gamma}.\end{aligned} \tag{C.5}$$

For the agent to be insolvent, it should hold that $u(\bar{b}-(1+r_s)s, l) \leq u(\underline{c}, 0)$, which requires

$$\bar{b} \leq (1+r_s)s + \frac{\phi}{1+\sigma}\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1+\sigma}{\sigma+\tau}} (z\rho)^{\frac{(1-\tau)(1+\sigma)}{\sigma+\tau}} + \underline{c}. \tag{C.6}$$

**Case 2: Solvency** When condition (C.6) is not satisfied, the agent is solvent if the highest wage rate is offered by job $\rho$. Therefore, the actual wage rate $w(\Omega, \rho, \rho')$ that solves the Nash bargaining problem should also satisfy the solvency condition, i.e.,

$$\begin{aligned}&(1+r)b + \varkappa\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1-\tau}{\sigma+\tau}} w(\Omega,\rho,\rho')^{\frac{(1+\sigma)(1-\tau)}{\sigma+\tau}} \\ &> (1+r_s)s + \frac{\phi}{1+\sigma}\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1+\sigma}{\sigma+\tau}} w(\Omega,\rho,\rho')^{\frac{(1-\tau)(1+\sigma)}{\sigma+\tau}} + \underline{c}.\end{aligned} \tag{C.7}$$

Otherwise, the agent would obtain a zero match surplus and choose not to supply labor, which also results in a zero match surplus for the firm. Thus both sides could be better off if the firm increases

consumes all wealth in the final period. In other periods $t < T$, due to the endogenous consumption and savings decisions, multiple iterations are needed to obtain the wage functions as fixed points.

the wage rate to satisfy the solvency condition (C.7). I now derive the wage function $w(\Omega, \rho, \rho')$ under condition (C.7).

The value for employed workers at age $T$ is:

$$W(\Omega, \rho, \rho') = \frac{1}{1-\gamma}\left[(1+r)b + \varkappa[w(\Omega,\rho,\rho')l]^{1-\tau} - (1+r_s)s - \phi\frac{l^{1+\sigma}}{1+\sigma}\right]^{1-\gamma}, \tag{C.8}$$

where $l = \left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1}{\sigma+\tau}} w(\Omega,\rho,\rho')^{\frac{1-\tau}{\sigma+\tau}}$.

The outside option value for employed workers with negotiation benchmark $\rho'$ at age $T$ is

$$\overline{W}(\Omega, \rho') = \max\left\{\frac{1}{1-\gamma}\left[(1+r)b + \varkappa(z\rho' l')^{1-\tau} - (1+r_s)s - \phi\frac{l'^{1+\sigma}}{1+\sigma}\right]^{1-\gamma}, \frac{\underline{c}^{1-\gamma}}{1-\gamma}\right\}, \tag{C.9}$$

where $l' = \left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1}{\sigma+\tau}} (z\rho')^{\frac{1-\tau}{\sigma+\tau}}$. The max operator considers the solvency/insolvency case at the negotiation benchmark $\rho'$.

The $w(\Omega, \rho, \rho')$ is chosen to maximize the bargaining objective function:

$$w(\Omega,\rho,\rho') = \underset{w(\Omega,\rho,\rho')}{\operatorname{argmax}} \left[W(\Omega,\rho,\rho') - \overline{W}(\Omega,\rho')\right]^{\xi} J(\Omega,\rho,\rho')^{1-\xi}. \tag{C.10}$$

Substituting equations (C.3), (C.8) and (C.9) into problem (C.10) and taking the first order condition, we obtain $w(\Omega, \rho, \rho')$ by solving the following root-finding problem:

$$\begin{aligned}
&\frac{\xi(1-\gamma)\left[B + Kw(\Omega,\rho,\rho')^{\frac{(1+\sigma)(1-\tau)}{\sigma+\tau}}\right]^{-\gamma} \varkappa(1-\tau)\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1-\tau}{\sigma+\tau}} w(\Omega,\rho,\rho')^{\frac{1-2\tau-\sigma\tau}{\sigma+\tau}}}{\left[B + Kw(\Omega,\rho,\rho')^{\frac{(1+\sigma)(1-\tau)}{\sigma+\tau}}\right]^{1-\gamma} - \left[\max\{B + K(z\rho')^{\frac{(1+\sigma)(1-\tau)}{\sigma+\tau}}, \underline{c}\}\right]^{1-\gamma}} \\
&= (1-\xi)\left[\frac{1}{z\rho - w(\Omega,\rho,\rho')} - \frac{1-\tau}{(\sigma+\tau)w(\Omega,\rho,\rho')}\right],
\end{aligned} \tag{C.11}$$

where $B = (1+r)b - (1+r_s)s$ and $K = \varkappa\frac{\sigma+\tau}{1+\sigma}\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1-\tau}{\sigma+\tau}}$.

I use bisection method to solve equation (C.11) with initial lower bound,

$$LB = \left[\frac{1+\sigma}{\varkappa(\sigma+\tau)}\left[\frac{\phi}{\varkappa(1-\tau)}\right]^{\frac{1-\tau}{\sigma+\tau}}\left[[(1-\gamma)\overline{W}(\Omega,\rho')]^{\frac{1}{1-\gamma}} - (1+r)b + (1+r_s)s\right]\right]^{\frac{\sigma+\tau}{(1+\sigma)(1-\tau)}}, \tag{C.12}$$

and upper bound,

$$UB = z\rho. \tag{C.13}$$

Finally, substituting the solution of $w(\Omega, \rho, \rho')$ into equations (C.3) and (C.8), we obtain $J(\Omega, \rho, \rho')$ and $W(\Omega, \rho, \rho')$.

Table C.4: Discretization of state space.

| Parameters | Value | Description |
|---|---|---|
| $n_b$ | 400 | Number of wealth grids |
| $\Delta_b$ | \$500 | Length of wealth grids |
| $[\underline{b} \quad \overline{b}]$ | $[\$0 \quad \$200,000]$ | Range of wealth |
| $n_s$ | 100 | Number of student loan debt grids |
| $\Delta_s$ | \$500 | Length of student loan debt grids |
| $[\underline{s} \quad \overline{s}]$ | $[\$0 \quad \$50,000]$ | Range of student loan debt |
| $n_\rho$ | 20 | Number of productivity grids |
| $\Delta_\rho$ | 0.05 | Length of productivity grids |
| $[\underline{\rho} \quad \overline{\rho}]$ | $[0 \quad 1]$ | Range of productivity |

### C.2.3 Implementation

To ensure accuracy, I choose relatively fine grids (see Table C.4), and the values between grids are approximated by linear interpolation. I use the golden section search method to find the optimal decision rules. The advantage of the golden section search method is that it is robust to the choice of initial values because convergence is guaranteed. However, convergence to the global optimum is not ensured if there are many local optima. Therefore, I further divide the whole decision space into multiple sub-space and select the largest local optimum. I do a robustness check after the estimation using a sequential grid search, and the results are identical. When solving the Nash bargaining problem, I need to invoke the calculation for utility from consumption and utility from the future multiple times. I save the computation time by calculating these values in advance and store them in memory.

The numerical algorithm is implemented using C++. The program is run on the server of MIT Economics Department, *supply.mit.edu*, which is built on Dell PowerEdge R910 running RedHat 6.7 (64-core processor, Intel(R) Xeon(R) CPU E7-4870, 2.4GHz). I use OpenMP for parallelization when iterating value functions and simulating the model. My baseline model requires 200GB of RAM to store the large number of decision rules and value functions.

# D Additional Theoretical Results

## D.1 Restructuring the Fixed Repayment Contract

The existence of the liquidity channel suggests that the lender can restructure the schedule of repayment to mitigate the debt burden. In reality, the federal student loan system has such features. For example, under the Direct Loan Program and the FFEL Program, borrowers have a 6-month grace period after graduation before payments are due. Moreover, the graduated repayment plan allows borrowers to make smaller payments at first and then increase their payments over time. The extended repayment plan allows qualified borrowers to extend the repayment period up to 25 years.

To formalize the intuition behind these realistic repayment plans, consider a particular contract that requires the agent to repay $s_1$ at $t = 1$, and $s_2$ at $t \geq 2$, such that the outstanding debt $S$ is recovered:

$$\frac{s_1}{1+r} + \sum_{t=2}^{\infty} \frac{s_2}{(1+r)^t} = S. \tag{D.1}$$

Proposition D.1 shows that back-loading debt payments increases the reservation wage at $t = 1$ through the liquidity channel.

**Proposition D.1.** *Reducing $s_1$ and increasing $s_2$ subject to the constraint (D.1) strictly increases the reservation wage at $t = 1$ when the borrowing constraint is binding.*

*Proof.* If the wage offer is accepted at $t = 1$, then the wage income becomes flat in the future. Therefore, the agent would perfectly smooth consumption by saving $s - s_1$ at $t = 1$, and consuming $w - s$ in every period. The value function is

$$W_1(w) = \frac{u(w-s)}{1-\beta}. \tag{D.2}$$

Under the twisted repayment schedule, suppose that the agent's borrowing constraint is binding when unemployed, i.e., the agent does not save at $t = 1$ if the wage offer is rejected. Then the value function is

$$U_1 = u(\theta - s_1) + \beta \int_{\theta}^{w_2^*} U_2 dF(w) + \beta \int_{w_2^*}^{\overline{w}} W_2(w) dF(w), \tag{D.3}$$

where $U_2$ and $W_2(w)$ are the value functions of rejecting and accepting the wage offer at $t = 2$ conditional on the wage offer being rejected at $t = 1$. $w_2^*$ is the reservation wage at $t = 2$; It is also the reservation wage for all $t > 2$ because the job search problem is stationary in later periods due to constant debt repayment and zero initial wealth. Therefore, we can write $U_2$ and $W_2(w)$ as

$$W_2(w) = \frac{u(w-s_2)}{1-\beta}. \tag{D.4}$$

$$U_2 = \frac{u(\theta - s_2)}{1-\beta} + \frac{\beta}{1-\beta} \int_{w_2^*}^{\overline{w}} [W_2(w) - U_2] dF(w). \tag{D.5}$$

The reservation wage at $t = 1$, $w_1^*$, is determined by

$$U_1 = W_1(w_1^*) \tag{D.6}$$

Substituting equations (D.2) and (D.3) into equation (D.6), we obtain

$$\frac{u(w_1^* - s)}{1-\beta} = u(\theta - s_1) + \beta \int_{\theta}^{w_2^*} U_2 dF(w) + \beta \int_{w_2^*}^{\overline{w}} W_2(w) dF(w). \tag{D.7}$$

Substituting equation (D.4) and $U_2 = W_2(w_2^*)$ into equation (D.7), we obtain

$$\frac{u(w_1^* - s)}{1-\beta} = u(\theta - s_1) + \frac{\beta}{1-\beta} \int_{\theta}^{w_2^*} u(w_2^* - s_2) dF(w) + \frac{\beta}{1-\beta} \int_{w_2^*}^{\overline{w}} u(w - s_2) dF(w). \tag{D.8}$$

Consider small changes of payments, $\Delta s_1 < 0$, equation (D.1) and assumption $\beta(1+r) = 1$ imply

$$\Delta s_2 = -r\Delta s_1 = -\frac{1-\beta}{\beta}\Delta s_1 > 0. \tag{D.9}$$

Differentiating equation (D.8):

$$\begin{aligned}\Delta w_1^* = &-\frac{1}{Q}u'(\theta - s_1)\Delta s_1 + \frac{\beta}{Q(1-\beta)}u'(w_2^* - s_2)F(w_2^*)\Delta w_2^* \\ &+ \frac{\beta}{Q(1-\beta)}\left[-u'(w_2^* - s_2) + \int_{w_2^*}^{\overline{w}}[u'(w_2^* - s_2) - u'(w - s_2)]dF(w)\right]\Delta s_2,\end{aligned} \tag{D.10}$$

where $Q = \frac{u'(w_1^* - s)}{1-\beta} > 0$.

The reservation wage at $t = 2$, $w_2^*$, is determined by $U_2 = W(w_2^*)$,

$$u(w_2^* - s_2) = u(\theta - s_2) + \frac{\beta}{1-\beta}\int_{w_2^*}^{\overline{w}}[u(w - s_2) - u(w_2^* - s_2)]dF(w). \tag{D.11}$$

Differentiating equation (D.11):

$$\Delta w_2^* = \frac{u'(w_2^* - s_2)\dfrac{1-\beta F(w_2^*)}{1-\beta} - u'(\theta - s_2) - \dfrac{\beta}{1-\beta}\displaystyle\int_{w_2^*}^{\overline{w}} u'(w - s_2)dF(w)}{u'(w_2^* - s_2)\dfrac{1-\beta F(w_2^*)}{1-\beta}}\Delta s_2. \tag{D.12}$$

Substituting (D.9) and (D.12) into (D.10), I obtain

$$\Delta w_1^* = -\frac{1}{Q}[u'(\theta - s_1) - \frac{(1-\beta)F(w_2^*)}{1-\beta F(w_2^*)}u'(\theta - s_2) - \frac{1}{1-\beta F(w_2^*)}\int_{w_2^*}^{\overline{w}} u'(w - s_2)dF(w)]\Delta s_1. \tag{D.13}$$

When the wage offer at $t = 1$ is rejected, the marginal utility of one unit of consumption at $t = 1$ is $u'(\theta - s_1)$, and the marginal utility of one unit of savings is

$$\beta\left[(1+r)u'(\theta - s_2)F(w_2^*) + \frac{r}{1-\beta}\int_{w_2^*}^{\overline{w}} u'(w - s_2)dF(w)\right]. \tag{D.14}$$

In (D.14), the first term captures that the agent would consume $(1+r)$ at marginal utility $u'(\theta - s_2)$ if the wage offer is below $w_2^*$ at $t = 2$ and rejected. The agent does not save in this case because debt payment is flat during $t \geq 2$ and expected income is higher. The second term captures that the agent would consume $r$ at marginal utility $u'(w - s_2)$ in every future period, $t \geq 2$, if the wage offer $w$ is above $w_2^*$ at $t = 2$ and accepted. This is because both wage income and debt payment are flat in every future period, $t \geq 2$. Thus the agent would only consume the interest of her one unit of wealth to perfectly smooth consumption.

The binding borrowing constraint implies that the marginal utility of one unit of consumption at

$t = 1$ is larger than the marginal utility of one unit of savings, i.e.,

$$u'(\theta - s_1) \geq F(w_2^*)u'(\theta - s_2) + \int_{w_2^*}^{\overline{w}} u'(w - s_2)dF(w). \tag{D.15}$$

Substituting (D.15) into (D.13),

$$\begin{aligned}
\Delta w_1^* &\geq -\frac{1}{Q}\left[\frac{\beta F(w_2^*)[1-F(w_2^*)]}{1-\beta F(w_2^*)}u'(\theta - s_2) - \frac{\beta F(w_2^*)}{1-\beta F(w_2^*)}\int_{w_2^*}^{\overline{w}} u'(w-s_2)dF(w)\right]\Delta s_1 \\
&= -\frac{\beta F(w_2^*)}{Q[1-\beta F(w_2^*)]}\left[[1-F(w_2^*)]u'(\theta - s_2) - \int_{w_2^*}^{\overline{w}} u'(w-s_2)dF(w)\right]\Delta s_1 \\
&= -\frac{\beta F(w_2^*)\Delta s_1}{Q[1-\beta F(w_2^*)]}\int_{w_2^*}^{\overline{w}}[u'(\theta - s_2) - u'(w-s_2)]dF(w) > 0 \qquad \text{(D.16)}
\end{aligned}$$

□

In contrast to Proposition 1, Proposition D.1 holds for any risk-averse agent, but it requires an imperfect credit market. When the borrowing constraint is not binding, the liquidity channel is absent because any change in the repayment schedule only results in a change in savings rather than affecting the job search decisions. When the borrowing constraint is binding, back-loading debt payments affects the reservation wage through two effects. First, reducing $s_1$ has a direct positive effect on the reservation wage at $t = 1$, because it provides liquidity for continued search. Second, reducing $s_1$ induces a higher $s_2$, resulting in a lower reservation wage at $t \geq 2$. The lower future reservation wages reduce the value of continued job search, which in turn indirectly imposes a negative effect on the reservation wage at $t = 1$. When the borrowing constraint is binding, the direct effect dominates the indirect effect. Intuitively, this is because the agent faces a higher marginal utility of consumption in the current period, thus she has the incentive to transfer wealth from future periods by setting a lower reservation wage. Requiring a smaller payment in the current period dampens this incentive by reducing the intertemporal gap in the marginal utility of consumption. As a result, the agent would increase her reservation wage to pursue a higher expected future return.

## D.2 Implication on Expected Income

A lower reservation wage implies that the agent would have less expected income when she is indebted under the fixed repayment contract. To see this, let $I(w_{FIX}^*)$ denote the present value of expected income as a function of the reservation wage $w_{FIX}^*$, and then it can be solved recursively:

$$I(w_{FIX}^*) = F(w_{FIX}^*)[\theta + \beta I(w_{FIX}^*)] + \int_{w_{FIX}^*}^{\overline{w}} \frac{w}{1-\beta}dF(w). \tag{D.17}$$

Equation (D.17) states that when the agent draws an offer below $w_{FIX}^*$ with probability $F(w_{FIX}^*)$, she rejects it and receives UI benefits $\theta$ in the current period and the same present value of expected income $I(w_{FIX}^*)$ in the next period. When the wage offer is above $w^*$, she accepts it and gets paid perpetually. The compensation for search risks implies a monotonic relationship between $w_{FIX}^*$ and $I(w_{FIX}^*)$:

**Proposition D.2.** *There exists a unique income-maximizing reservation wage $\hat{w}$, determined by*

$$\hat{w} - \frac{\beta}{1-\beta}\int_{\hat{w}}^{\overline{w}}(w-\hat{w})dF(w) = \theta. \tag{D.18}$$

*The present value of expected income is strictly increasing in $w^*_{FIX}$ when $w^*_{FIX} < \hat{w}$, and strictly decreasing in $w^*_{FIX}$ when $w^*_{FIX} > \hat{w}$. Moreover, the optimal reservation wage for any risk-averse agent satisfies $w^*_{FIX} < \hat{w}$.*

*Proof.* Rearranging equation (D.17),

$$I(w^*_{FIX}) = \frac{\theta F(w^*_{FIX}) + \frac{1}{1-\beta}\int_{w^*_{FIX}}^{\overline{w}} wdF(w)}{1-\beta F(w^*_{FIX})}. \tag{D.19}$$

Take the first derivative,

$$I'(w^*_{FIX}) = \frac{f(w^*_{FIX})}{[1-\beta F(w^*_{FIX})]^2}\left[\theta - w^*_{FIX} + \frac{\beta}{1-\beta}\int_{w^*_{FIX}}^{\overline{w}}(w-w^*_{FIX})dF(w)\right]. \tag{D.20}$$

Denote

$$h(x) = \theta - x + \frac{\beta}{1-\beta}\int_{x}^{\overline{w}}(w-x)dF(w). \tag{D.21}$$

It is straightforward to show that $h(\theta) > 0$, $h(\overline{w}) < 0$, and $h(x)' < 0$. Thus there exists a unique $w^*_{FIX} \in (\theta, \overline{w})$, denoted as $\hat{w}$, such that $I'(\hat{w}) = 0$. When $w^* < \hat{w}$, $I'(w^*_{FIX}) > 0$ and expected income is strictly increasing in $w^*_{FIX}$; when $w^*_{FIX} > \hat{w}$, $I'(w^*_{FIX}) < 0$ and expected income is strictly decreasing in $w^*_{FIX}$. Therefore, $\hat{w}$ maximizes expected income and is determined by

$$\hat{w} - \frac{\beta}{1-\beta}\int_{\hat{w}}^{\overline{w}}(w-\hat{w})dF(w) = \theta. \tag{D.22}$$

Now, I prove that a risk-neutral agent sets her reservation wage to be $\hat{w}$. Because the interest rate is assumed to satisfy $\beta(1+r) = 1$, the risk-neutral agent is indifferent about savings. Without loss of generality, I assume that the risk-neutral agent also behaves hand-to-mouth, like a risk-averse agent. Therefore, her reservation wage is determined by equation (3.3).

The utility function of the risk-neutral agent has a linear form, i.e., $u(x) = ax + b$. Substituting this into equation (3.3), I obtain

$$w^*_{FIX} - \frac{\beta}{1-\beta}\int_{w^*_{FIX}}^{\overline{w}}(w-w^*_{FIX})dF(w) = \theta. \tag{D.23}$$

There is a unique solution to equation (D.23), thus $w^*_{FIX} = \hat{w}$ for the risk-neutral agent. ☐

In fact, the income-maximizing reservation wage $\hat{w}$ is the reservation wage set by risk-neutral agents. In an incomplete market, the existence of uninsured search risks incentivizes risk-averse agents to set a strictly lower reservation wage in order to smooth consumption.

### D.3 Tradeoff Between Insurance and Incentive to Work

Because IBR provides insurance, it is not surprising that it increases welfare relative to the fixed repayment contract. I prove this result under CRRA utility:

**Proposition D.3.** *With CRRA utility (and inelastic labor supply), IBR improves the agent's welfare relative to the fixed repayment contract.*

*Proof.* The welfare of the agent under the two repayment contracts is given by:

$$\text{Welfare}_{IBR} = \frac{F(w^*_{IBR})u((1-\alpha)\theta)}{1-\beta F(w^*_{IBR})} + \int_{w^*_{IBR}}^{\overline{w}} \frac{u((1-\alpha)w)}{(1-\beta)[1-\beta F(w^*_{IBR})]} dF(w);$$

$$\text{Welfare}_{FIX} = \frac{F(w^*_{FIX})u(\theta - s)}{1-\beta F(w^*_{FIX})} + \int_{w^*_{FIX}}^{\overline{w}} \frac{u(w-s)}{(1-\beta)[1-\beta F(w^*_{FIX})]} dF(w). \tag{D.24}$$

Notice that

$$\frac{(1-\beta)F(w^*) + \int_{w^*}^{\overline{w}} dF(w)}{1-\beta F(w^*)} = 1, \tag{D.25}$$

which allows us to define a CDF $G(w; w^*)$ with parameter $w^*$ as follows:

$$G(w; w^*) = \begin{cases} \dfrac{(1-\beta)F(w^*)}{1-\beta F(w^*)} & \text{if } w \in [\theta, w^*], \\ \dfrac{F(w) - \beta F(w^*)}{1-\beta F(w^*)} & \text{if } w \in (w^*, \overline{w}]. \end{cases} \tag{D.26}$$

Then the welfare equations (D.24) can be written as:

$$\text{Welfare}_{IBR} = \frac{1}{1-\beta} \int_{\theta}^{\overline{w}} u((1-\alpha)w) dG(w; w^*_{IBR}); \tag{D.27}$$

$$\text{Welfare}_{FIX} = \frac{1}{1-\beta} \int_{\theta}^{\overline{w}} u(w-s) dG(w; w^*_{FIX}). \tag{D.28}$$

To prove that $\text{Welfare}_{IBR} > \text{Welfare}_{FIX}$, it is sufficient to show that the lottery with value $(1-\alpha)w$ and CDF $G(w; w^*_{IBR})$ is second-order stochastically dominant over the lottery with value $w - s$ and CDF $G(w; w^*_{FIX})$.

Following the proof of Lemma 1, the single-crossing condition is satisfied because $(1-\alpha)\theta > \theta - s$. Thus I only need to show that the mean of lottery $G(w; w^*_{IBR})$ is larger than the mean of lottery $G(w; w^*_{FIX})$:

$$\int_{\theta}^{\overline{w}} (1-\alpha) w dG(w; w^*_{IBR}) > \int_{\theta}^{\overline{w}} (w-s) dG(w; w^*_{FIX}). \tag{D.29}$$

Define $L(w^*)$ as follows:

$$L(w^*) = \int_{\theta}^{\overline{w}} w dG(w; w^*) = \frac{(1-\beta)\theta F(w^*) + \int_{w^*}^{\overline{w}} w dF(w)}{1-\beta F(w^*)}. \tag{D.30}$$

Taking the first derivative w.r.t. $w^*$:

$$L(w^*)' = \frac{(1-\beta)f(w^*)}{[1-\beta F(w^*)]^2}\left[\theta - w^* + \frac{\beta}{1-\beta}\int_{w^*}^{\overline{w}}(w-w^*)dF(w)\right]. \tag{D.31}$$

According to the proof of Proposition D.2, $L(w^*)' > 0$ as long as $w^* < \hat{w}$, which is always the case because $\hat{w}$ is the reservation wage chosen by a risk-neutral agent.

Proposition 2 shows that with CRRA utility $w^*_{IBR} > w^*_{FIX}$. Then $L(w^*)' > 0$ implies that

$$\int_{\theta}^{\overline{w}} wdG(w; w^*_{IBR}) > \int_{\theta}^{\overline{w}} wdG(w; w^*_{FIX}). \tag{D.32}$$

The repayment ratio $\alpha$ is determined by equations (D.17) and (3.4), thus

$$s = \frac{(1-\beta)F(w^*_{IBR})\alpha\theta + \int_{w^*_{IBR}}^{\overline{w}} \alpha w dF(w)}{1-\beta F(w^*_{IBR})} = \int_{\theta}^{\overline{w}} \alpha w dG(w; w^*_{IBR}). \tag{D.33}$$

This implies

$$\int_{\theta}^{\overline{w}} \alpha w dG(w; w^*_{IBR}) = \int_{\theta}^{\overline{w}} sdG(w; w^*_{FIX}). \tag{D.34}$$

Equations (D.32) and (D.34) lead to (D.29). □

However, this proposition may not hold if labor supply is sufficiently elastic, because IBR also naturally introduces an income-tax-like distortion, resulting in an efficiency loss. Moreover, the proof of Proposition D.3 also hinges on Proposition 2.[6] This implies that the reservation wage effect of income contingency plays a role in determining the agent's welfare.

In the following, I introduce elastic labor supply to elucidate the tradeoff between the two contracts and the implication of the reservation wage effect on welfare. To this end, I consider a simple mix of the two contracts by assuming that the lender is restricted to using a linear combination of IBR and the fixed repayment contract.[7]

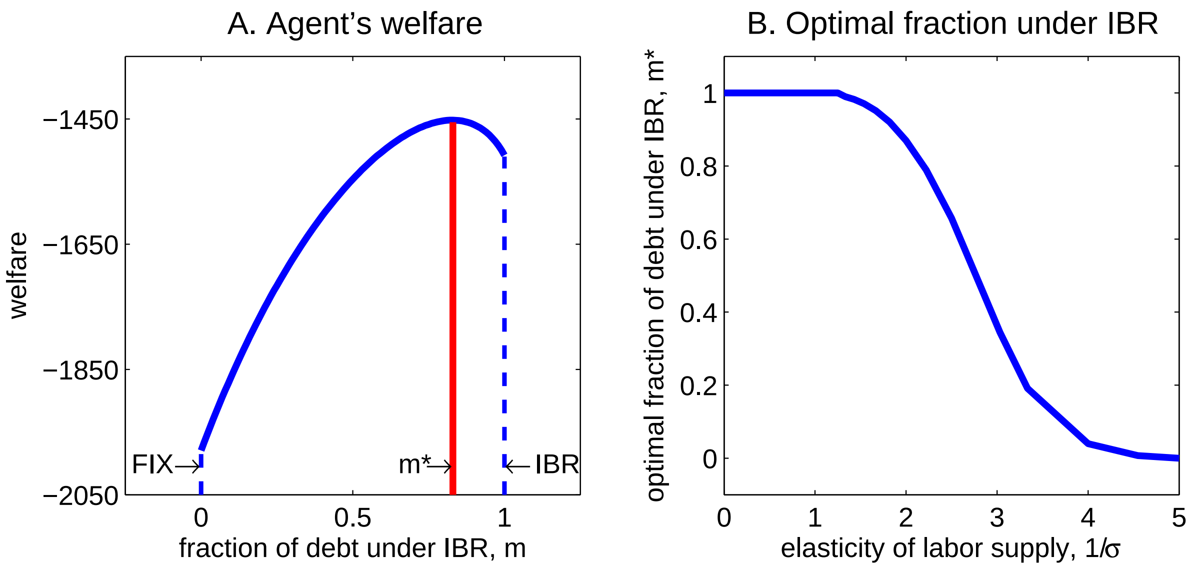
In particular, the lender makes a fraction of debt $mS$ income contingent, and the rest $(1-m)S$ is repaid under the fixed repayment contract, where $m \in [0, 1]$. Under this linear contract, in each period the agent repays

$$s = \begin{cases} \alpha\theta + r(1-m)S & \text{if unemployed,} \\ \alpha z + r(1-m)S & \text{if employed with earnings } z = wl(w, \alpha), \end{cases} \tag{D.35}$$

where labor supply $l(w, \alpha)$ is a function of the wage rate $w$ and the repayment ratio $\alpha$.

[6] For a general DARA utility, the proof is not obtained because it is not clear whether IBR raises the reservation wage.

[7] I consider this linear contract for its simplicity and transparency to illustrate the idea. It is also partially motivated by the numerical examples of Mirrlees (1971) that the optimal tax schedule is hardly different from an affine function with a constant marginal tax rate. However, numerical simulations from later research show that optimal tax schedules are very sensitive to the utility functions and income distributions.




Note: This figure illustrates the tradeoff between insurance and the incentive to work. I consider a simple mix of the two contracts by assuming that the lender is restricted to using a linear combination of IBR and the fixed repayment contract. Panel A plots the agent's welfare when the fraction of debt repaid under IBR varies from zero (corresponding to the pure fixed repayment contract) to one (corresponding to the pure IBR). It shows that the agent's welfare first increases then decreases due to the benefit from insurance and the distortion on labor supply. The optimal fraction under this parametrization is given by an interior point $m^*$. Panel B plots the optimal fraction of debt under IBR when the elasticity of labor supply varies. A more elastic labor supply increases the distortion on labor supply, thus making IBR less desirable. The figure is plotted using the GHH utility, $u(c,l) = [c - \phi l^{1+\sigma}/(1+\sigma)]^{1-\gamma}/(1-\gamma)$, and beta distribution of wage offers, $Beta(a,b)$, with parameter values: $a = 2$, $b = 4$, $\gamma = 3$, $\theta = 0.1$, $\overline{w} = 1.1$, $\beta = 0.96$, $S = 1$, $\phi = 1$, $\sigma = 0.47$.

Figure D.1: A numerical illustration of the agent's welfare and the optimal fraction of debt repaid under IBR.

For the lender to break even, the repayment ratio $\alpha$ is chosen to satisfy the recoverability constraint[8],

$$\frac{mD}{\beta} = \frac{F(w^*)}{1-\beta F(w^*)}\alpha\theta + \frac{\alpha}{(1-\beta)[1-\beta F(w^*)]}\int_{w^*}^{\overline{w}} wl(w,\alpha)dF(w). \tag{D.36}$$

I use GHH utility (Greenwood, Hercowitz and Huffman, 1988), $u(c,l) = \frac{1}{1-\gamma}\left(c - \phi\frac{l^{1+\sigma}}{1+\sigma}\right)^{1-\gamma}$, to provide several numerical examples. Panel A of Figure D.1 shows that depending on parameter values, increasing $m$ may increase or decrease the agent's welfare due to the the tradeoff in insurance and the incentive to work. The optimal fraction of debt repaid under IBR $m^*$ that maximizes the agent's welfare could be an interior point. Intuitively, there are diminishing returns in providing insurance through IBR due to the decreasing marginal utility of consumption. On the other hand, the distortion on labor supply increases as a higher $m$ increases the repayment ratio $\alpha$. The optimal value of $m^*$ is achieved when the marginal benefit from providing insurance is equal to the marginal cost of labor supply distortion.

In general, $m^*$ could also be a corner solution, in which case the full IBR is strictly better than the fixed repayment contract or vice versa. Panel B of Figure D.1 indicates that whether IBR results in a higher welfare crucially depends on the elasticity of labor supply. This is because the elasticity of labor supply determines how responsive labor supply would be when a fraction of income is extracted by

[8]Because earnings depend on labor supply, which is a function of $\alpha$, there is a Laffer curve for expected debt repayment, and there may not exist a solution to equation (D.36) when the debt level is high. My following numerical analyses consider the case in which there exist solutions to equation (D.36) and the smaller $\alpha$ is always selected.

the lender. When labor supply is completely inelastic, IBR is strictly better as shown in Proposition D.3. However, when labor supply is very elastic, the distortion on labor supply is large; so the fixed repayment contract results in a higher welfare.

In Figure D.2, I illustrate that the insurance provided by IBR is more valuable due to the positive response in the reservation wage. In particular, I gradually increase $m$ from 0 to 1 and compare the change in welfare and expected labor supply in two scenarios. In one scenario, I allow the agent to endogenously choose the reservation wage; in the other scenario, I fix the reservation wage at the beginning. Because IBR raises the reservation wage, the reservation wage in the first scenario is increasing as $m$ increases (see panel A of Figure D.2).

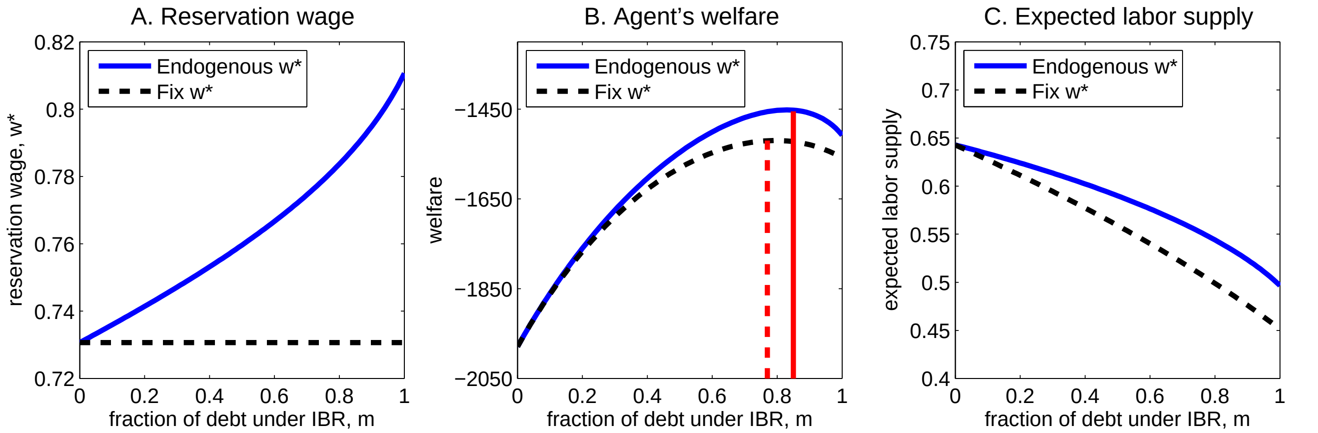
Panel B of Figure D.2 indicates that the welfare is significantly higher in the first scenario. This illustrates that IBR increases welfare not only by directly providing insurance, but also by indirectly increasing the reservation wage. In other words, the insurance provided by IBR is more desirable when there are search risks because the agent would choose a higher reservation wage when search risks are partially insured. As a result, the optimal fraction of debt, $m^*$, is also higher in the first scenario.

Note that this result is not general and would be violated if the elasticity of labor supply is very large. In fact, when labor supply is elastic, IBR raises the reservation wage through an additional channel. This is because repaying debt as a fraction of income disproportionately reduces income more during employment relative to during unemployment because of the negative response in labor supply. This generates a "debt overhang" effect. The "debt overhang" channel not only reduces labor supply, but also further incentivizes the agent to set a higher reservation wage in order to stay unemployed.[9] When the elasticity of labor supply is sufficiently large, the reservation wage could be higher than the efficient one; as a result, fixing the reservation wage at some lower level could be welfare improving. See Appendix D.4 for a detailed discussion.

Panel C compares the expected labor supply in the two scenarios. It shows that the negative effect on labor supply is smaller when the reservation wage is endogenous. This is due to two channels: first, there is a direct positive substitution effect on labor supply as IBR increases the average wage rate by raising the reservation wage. Second, there is an indirect effect due to a lower repayment ratio. This is because a higher reservation wage increases expected repayment conditional on any repayment ratio. Therefore, when the reservation wage increases, the lender would set a lower repayment ratio according to the recoverability constraint (3.4). This in turn alleviates the distortion on labor supply.

These numerical examples suggest that the positive response in the reservation wage under IBR offers a channel that not only increases the agent's welfare but also alleviates the distortionary effect on labor supply. These results highlight that despite the canonical tradeoff between insurance and the incentive to work, IBR is in fact more valuable compared to the fixed repayment contract because of uninsured search risks.

[9] I would like to highlight the distinction between the three channels: the risk channel, the liquidity channel, and the debt overhang channel. Although all three channels raise the reservation wage under IBR, they have divergent welfare implications. The increase in the reservation wage through the risk channel and the liquidity channel is a beneficial response to the correction of the credit and insurance market failures. However, the increase in the reservation wage through the debt overhang channel is a sub-optimal response to the distortion in the relative price of employment and unemployment.




Note: This figure illustrates the reservation wage effect. The blue solid line plots the agent's reservation wage, welfare, and expected labor supply when the reservation wage is allowed to increase as a larger fraction of debt is made income contingent. The black dashed line plots the agent's reservation wage, welfare, and expected labor supply when the reservation wage is fixed at the initial value under the pure fixed repayment contract (see panel A). Panel B shows that the agent's welfare is higher if the reservation wage is allowed to increase; as a result, the optimal fraction of debt repaid under IBR is also larger. Panel C shows that the reduction in labor supply also becomes smaller due to the higher reservation wage. The figure is plotted using the GHH utility, $u(c,l) = [c - \phi l^{1+\sigma}/(1+\sigma)]^{1-\gamma}/(1-\gamma)$, and the beta distribution of wage offers, $Beta(a,b)$, with parameter values: $a = 2$, $b = 4$, $\gamma = 3$, $\theta = 0.1$, $\overline{w} = 1.1$, $\beta = 0.96$, $S = 1$, $\phi = 1$, $\sigma = 0.47$.

Figure D.2: A numerical illustration of the reservation wage effect.

## D.4 Understanding the Reservation Wage Effect

In this appendix section, I discuss the conditions under which the reservation wage effect of IBR raises the agent's welfare.

In Figure D.2, I provided a numerical example showing that IBR also indirectly increases welfare by increasing the reservation wage (i.e., the reservation wage effect). While this result holds for a wide range of empirically reasonable parameter values, it is not generally true. I now elucidate the economic intuitions.

In subsection D.4.1, I characterize the efficient IBR under the assumption that the reservation wage is observable and contractible. I define the reservation wage set by this contract as the efficient reservation wage. In subsection D.4.2, I show that when labor supply is inelastic, the reservation wage under IBR is below the efficient reservation wage. This explains why the reservation wage effect increases welfare. In subsection D.4.3, I show that when labor supply is sufficiently elastic, the reservation wage under IBR could be above the efficient reservation wage. This is because there is an additional debt overhang channel under IBR that further increases the reservation wage. The implication of this is that the reservation wage effect could reduce welfare. Finally, in subsection D.4.4, I provide several numerical examples and discuss that this counter-intuitive result is not likely to happen in reality. Therefore, I argue that IBR indirectly increases welfare by increasing the reservation wage.

### D.4.1 Efficient Reservation Wage

For a certain reservation wage $w^*$, the agent's welfare under IBR can be expressed recursively:

$$\text{Welfare}_{IBR}(w^*) = F(w^*)\left[u((1-\alpha)\theta, 0) + \beta \text{Welfare}_{IBR}(w^*)\right] + \int_{w^*}^{\overline{w}} \frac{u((1-\alpha)wl, l)}{1-\beta} dF(w). \tag{D.37}$$

Thus, the agent's welfare is

$$\text{Welfare}_{IBR}(w^*) = \frac{F(w^*)u((1-\alpha)\theta, 0)}{1-\beta F(w^*)} + \int_{w^*}^{\overline{w}} \frac{u((1-\alpha)wl, l)}{(1-\beta)[1-\beta F(w^*)]} dF(w). \tag{D.38}$$

The agent determines the reservation wage $w^*_{IBR}$ to maximize welfare under IBR:

$$\begin{aligned} \max_{w^*} \quad & \text{Welfare}_{IBR}(w^*) & & , \\ \text{subject to} \quad & (1-\alpha)wu_1((1-\alpha)wl, l) + u_2((1-\alpha)wl, l) & = \quad & 0, \forall w \in [w^*, \overline{w}], \end{aligned} \tag{D.39}$$

where the constraint is the intra-temporal Euler equation on labor supply, $l(w, \alpha)$. If labor supply is inelastic, the solution to problem (D.39) gives the indifference equation (3.5). Conditional on the reservation wage that solves problem (D.39), the lender sets the repayment ratio $\alpha$ according to the recoverability constraint:

$$\frac{F(w^*_{IBR})\alpha\theta}{1-\beta F(w^*_{IBR})} + \int_{w^*_{IBR}}^{\overline{w}} \frac{\alpha wl(w,\alpha)}{(1-\beta)[1-\beta F(w^*_{IBR})]} dF(w) = \frac{S}{\beta}. \tag{D.40}$$

The reservation wage $w^*_{IBR}$ is inefficient because the agent's reservation wage generates an externality on the lender's revenue. The agent would be better off if she can internalize this effect when choosing the reservation wage. For the discussion of the reservation wage effect, it is useful to introduce the efficient reservation wage as a benchmark.

**Definition 1.** *The efficient reservation wage, $w^*_{EFI}$, is the reservation wage that the lender would set under IBR if the reservation wage is observable and contractible, i.e., $w^*_{EFI}$ solves:*

$$\begin{aligned} \max_{w^*} \quad & \text{Welfare}_{IBR}(w^*) & & \\ \text{subject to} \quad & (1-\alpha)wu_1((1-\alpha)wl, l) + u_2((1-\alpha)wl, l) & = \quad & 0, \forall w \in [w^*, \overline{w}], \\ & \frac{F(w^*)\alpha\theta}{1-\beta F(w^*)} + \int_{w^*}^{\overline{w}} \frac{\alpha wl(w,\alpha)}{(1-\beta)[1-\beta F(w^*)]} dF(w) & = \quad & \frac{S}{\beta}, \end{aligned} \tag{D.41}$$

*where the first constraint is the intra-temporal Euler equation on labor supply, and the second constraint is the lender's recoverability constraint.*

Clearly, $w^*_{EFI}$ is different from $w^*_{IBR}$ as the agent takes into account the lender's recoverability constraint when setting the reservation wage.

### D.4.2 Inelastic Labor Supply

To provide some intuitions, I begin by discussing the reservation wage effect when the agent has inelastic labor supply.

Suppose that the agent has CRRA utility, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. Denote $\lambda$ as the Lagrangian multiplier for the recoverability constraint in problem (D.41). The shadow price $\lambda$ is negative as the agent's welfare decreases when debt $S$ marginally increases. The first order condition that determines the efficient reservation wage is:

$$\frac{[(1-\alpha)\theta]^{1-\gamma}}{[1-\beta F(w^*_{EFI})](1-\gamma)} + \int_{w^*_{EFI}}^{\overline{w}} \frac{\beta[(1-\alpha)w]^{1-\gamma}}{(1-\beta)[1-\beta F(w^*_{EFI})](1-\gamma)} dF(w) - \frac{[(1-\alpha)w^*_{EFI}]^{1-\gamma}}{(1-\beta)(1-\gamma)}$$
$$= \lambda \frac{1-\beta F(w^*_{EFI})}{f(w^*_{EFI})} \alpha I'(w^*_{EFI}), \tag{D.42}$$

where $I'(w^*_{EFI})$ is the first derivative of expected income with respect to the reservation wage, characterized by equation (D.20). The RHS of equation (D.42) captures the effect of the reservation wage on expected repayment.

Define

$$g(x) = \frac{[(1-\alpha)\theta]^{1-\gamma}}{1-\gamma} + \int_{x}^{\overline{w}} \frac{\beta[(1-\alpha)w]^{1-\gamma}}{(1-\beta)(1-\gamma)} dF(w) - \frac{[1-\beta F(x)][(1-\alpha)x]^{1-\gamma}}{(1-\beta)(1-\gamma)}, \tag{D.43}$$
$$h(x) = \lambda \frac{[1-\beta F(x)]^2}{f(x)} \alpha I'(x). \tag{D.44}$$

Equation (D.42) can be rewritten as

$$g(w^*_{EFI}) - h(w^*_{EFI}) = 0. \tag{D.45}$$

In fact, $g(x) = 0$ coincides with the indifference equation (3.5), thus the solution to $g(x) = 0$ gives the reservation wage under IBR, i.e., $g(w^*_{IBR}) = 0$.

The proof of Proposition 2 indicates that with CRRA utility $w^*_{IBR} = w^*|_{s=0}$. When the agent is risk averse, according to Proposition D.2, $w^*|_{s=0} < \hat{w}$ and $I'(w^*_{IBR}) > 0$. With $\lambda < 0$, we have $h(w^*_{IBR}) < 0$. Thus

$$g(w^*_{IBR}) - h(w^*_{IBR}) > 0. \tag{D.46}$$

Take the first derivative for $g(x)$ and $h(x)$, we obtain:

$$g'(x) = -\frac{(1-\alpha)[1-\beta F(x)]}{1-\beta}[(1-\alpha)x]^{-\gamma} < 0, \tag{D.47}$$
$$h'(x) = -\frac{\lambda\alpha}{1-\beta}[1-\beta F(x)] > 0. \tag{D.48}$$

Thus

$$g'(x) - h'(x) < 0. \tag{D.49}$$

Equations (D.45-D.49) imply $w^*_{IBR} < w^*_{EFI}$. Therefore, the agent's efficient reservation wage is higher than the reservation wage under IBR when labor supply is inelastic. Intuitively, this is because the efficient reservation wage internalizes the choice of the reservation wage on expected repayment. By increasing the reservation wage, the agent could increase the lender's revenue, motivating the lender to set a smaller repayment ratio $\alpha$ given the recoverability constraint, which in turn increases welfare. The efficient reservation wage is not incentive compatible because facing a lower repayment ratio ex-post, the agent would have the incentive to reduce the reservation wage in order to take fewer risks and increase her utility. As a result, the lender would take a loss.

What this implies is that IBR indirectly raises the agent's welfare by increasing the reservation wage. If we restrict the agent from choosing a higher reservation wage, as in the experiment of Figure D.2, the agent's welfare would be lowered because the reservation wage is further away from the efficient one.

### D.4.3 Elastic Labor Supply

Now I turn to the discussion of the reservation wage effect when the agent has elastic labor supply. I show that with elastic labor supply, there is an additional channel that increases the reservation wage under IBR. As a result, the agent could possibly choose a reservation wage higher than the efficient reservation wage.

To illustrate the economic channel, I begin my analysis with risk-neutral agents. Suppose that the agent has quasi-linear utility $u(c,l) = c - \frac{l^{1+\sigma}}{1+\sigma}$. Using equations (3.3) and (3.5), $w^*_{FIX}$ and $w^*_{IBR}$ can be derived from:

$$\frac{\sigma}{1+\sigma}\left(w^*_{FIX}\right)^{\frac{1+\sigma}{\sigma}} = \theta + \frac{\sigma}{1+\sigma}\frac{\beta}{1-\beta}\int_{w^*_{FIX}}^{\overline{w}}\left[w^{\frac{1+\sigma}{\sigma}} - \left(w^*_{FIX}\right)^{\frac{1+\sigma}{\sigma}}\right]dF(w), \tag{D.50}$$

$$\frac{\sigma}{1+\sigma}\left(w^*_{IBR}\right)^{\frac{1+\sigma}{\sigma}} = \underbrace{(1-\alpha)^{-\frac{1}{\sigma}}}_{\text{debt overhang channel}} \theta + \frac{\sigma}{1+\sigma}\frac{\beta}{1-\beta}\int_{w^*_{IBR}}^{\overline{w}}\left[w^{\frac{1+\sigma}{\sigma}} - \left(w^*_{IBR}\right)^{\frac{1+\sigma}{\sigma}}\right]dF(w). \tag{D.51}$$

The only difference between the two equations lies in the term $(1-\alpha)^{-\frac{1}{\sigma}} > 1$, due to the response in labor supply when the agent is employed and repaying debt under IBR. As a result, the reservation wage under IBR is higher than that under the fixed repayment contract when $\sigma < \infty$. Note that Proposition D.2 implies that under the fixed repayment contract, the risk-neutral agent sets the reservation wage equal to $\hat{w}$, which already maximizes expected income. However, IBR further raises the reservation wage, which reduces expected income (before repayment). Intuitively, the agent chooses to set a higher reservation wage to avoid employment because supplying labor is costly. Therefore, elastic labor supply generates an additional force that increases the reservation wage under IBR. This channel is exposed starkly when the agent is risk neutral, because with inelastic labor supply ($\sigma = \infty$), the two reservation wages are equalized, $w^*_{FIX} = w^*_{IBR} = \hat{w}$, due to the absence of the risk channel and the liquidity channel discussed in subsection 3.2.1.

I name the effect on the reservation wage introduced by the elastic labor supply as the *debt overhang channel* of IBR.[10] I would like to highlight the distinction between the three channels: the debt overhang channel, the risk channel, and the liquidity channel. Although all three channels raise the reservation wage under IBR, they have divergent welfare implications. The increase in the reservation wage through the risk channel and the liquidity channel is a beneficial response to the correction of the credit and insurance market failures. However, the increase in the reservation wage through the debt overhang channel is a sub-optimal response to the distortion in the relative price of employment and

[10] This channel is related to the moral hazard problem in the labor market associated with debt collection policies (Mulligan, 2009).

unemployment.[11] Because the reservation wage controls the extensive participation margin of labor supply, we can interpret this result in an alternative way: IBR generates a moral hazard problem that reduces labor supply on the intensive margin. This in turn generates a moral hazard problem that reduces labor supply on the extensive margin, i.e., increasing the reservation wage.

The discussion above suggests that the reservation wage under IBR could be larger than the efficient reservation wage when the risk-neutral agent has elastic labor supply. To see this, I substitute the utility function into equation (D.38) and obtain the agent's welfare:

$$(1-\alpha)\left[\frac{F(w^*)}{1-\beta F(w^*)}\theta+\frac{\sigma}{1+\sigma}\frac{(1-\alpha)^{\frac{1}{\sigma}}}{(1-\beta)[1-\beta F(w^*)]}\int_{w^*}^{\overline{w}} w^{\frac{1+\sigma}{\sigma}}dF(w)\right]. \tag{D.52}$$

By substituting the expression for labor supply, $l=[(1-\alpha)w]^{1/\sigma}$, into equation (D.40), we obtain the recoverability constraint:

$$\alpha\left[\frac{F(w^*)}{1-\beta F(w^*)}\theta+\frac{(1-\alpha)^{\frac{1}{\sigma}}}{(1-\beta)[1-\beta F(w^*)]}\int_{w^*}^{\overline{w}} w^{\frac{1+\sigma}{\sigma}}dF(w)\right]=\frac{S}{\beta}. \tag{D.53}$$

The reservation wage $w^*_{IBR}$ is chosen to maximize the objective function (D.52) with the repayment ratio $\alpha$ set separately according to equation (D.53). The efficient reservation wage $w^*_{EFI}$ is chosen to maximize the objective function (D.52) subject to the constraint (D.53). It is clear that when $\sigma=\infty$, the reservation wage that maximizes the objective function (D.52) also simultaneously maximizes expected repayment, i.e., the LHS of equation (D.53). This implies that the first-order derivative of equation (D.53) with respect to the reservation wage is equal to zero. Therefore, the unconstrained maximization problem yields the same solution as the constrained maximization problem, i.e., $w^*_{IBR}=w^*_{EFI}$. Intuitively, this is saying that the risk-neutral agent would choose the efficient reservation wage that maximizes expected repayment when labor supply is inelastic.

However, when $\sigma<\infty$, the terms inside the bracket of (D.52) differ from those of (D.53) as less weight is given for the value of employment ($\frac{\sigma}{1+\sigma}<1$).[12] This suggests that, compared with the efficient reservation wage $w^*_{EFI}$ that solves the constrained maximization problem, the unconstrained maximization would set a relatively higher reservation wage $w^*_{IBR}$ to avoid employment.

The analysis of a risk-neutral agent presents the stark result that the reservation wage under IBR is always higher than the efficient reservation wage as long as labor supply is elastic. When the agent is risk averse, the risk and liquidity channels of debt repayment would reduce the reservation wage. Therefore, whether the reservation wage under IBR is higher than the efficient one depends on which channel dominates. Intuitively, the strength of the debt overhang channel increases with the elasticity of labor supply. Therefore, when labor supply is sufficiently elastic, the debt-overhang channel would

[11] Due to the response in labor supply, IBR essentially subsidizes unemployment by reducing income during employment by a proportion, $1-(1-\alpha)^{\frac{1+\sigma}{\sigma}}$, larger than the proportional reduction during unemployment, $\alpha$.

[12] Intuitively, the agent puts less weight on the value of employment in the objective function because supplying labor generates a dis-utility equaling to $\frac{1}{1+\sigma}$ of the agent's wage income. Mathematically, the efficient reservation wage that solves the constrained maximization problem can be thought of as the average of the reservation wage maximizing (D.52) and the one maximizing (D.52) weighted by the Lagrangian multiplier. Due to the existence of the term $\frac{\sigma}{1+\sigma}$ in (D.52), the reservation wage that maximizes (D.52) is higher than the one maximizing (D.53).

dominate and the reservation wage under IBR would be inefficiently high.[13]

The implication of the debt-overhang channel is that the agent could be better off if the reservation wage is restricted at some lower value when being provided with IBR. Therefore, it is not generally true that IBR also indirectly raises welfare by increasing the reservation wage.

#### D.4.4 Numerical Examples and Discussions

In this subsection, I provide numerical examples by setting different values for the elasticity of labor supply. The goal of this simple exercise is to show that for empirically reasonable values of risk aversion and the elasticity of labor supply, IBR increases welfare by raising the reservation wage.

In Figure D.3, I report the agent's reservation wage and welfare for different values of parameter $\sigma$. In each panel, I vary the fraction of debt under IBR and plot the outcome of interest when the reservation wage is endogenous, fixed at its value under the fixed repayment contract (i.e., $m = 0$), or efficient.

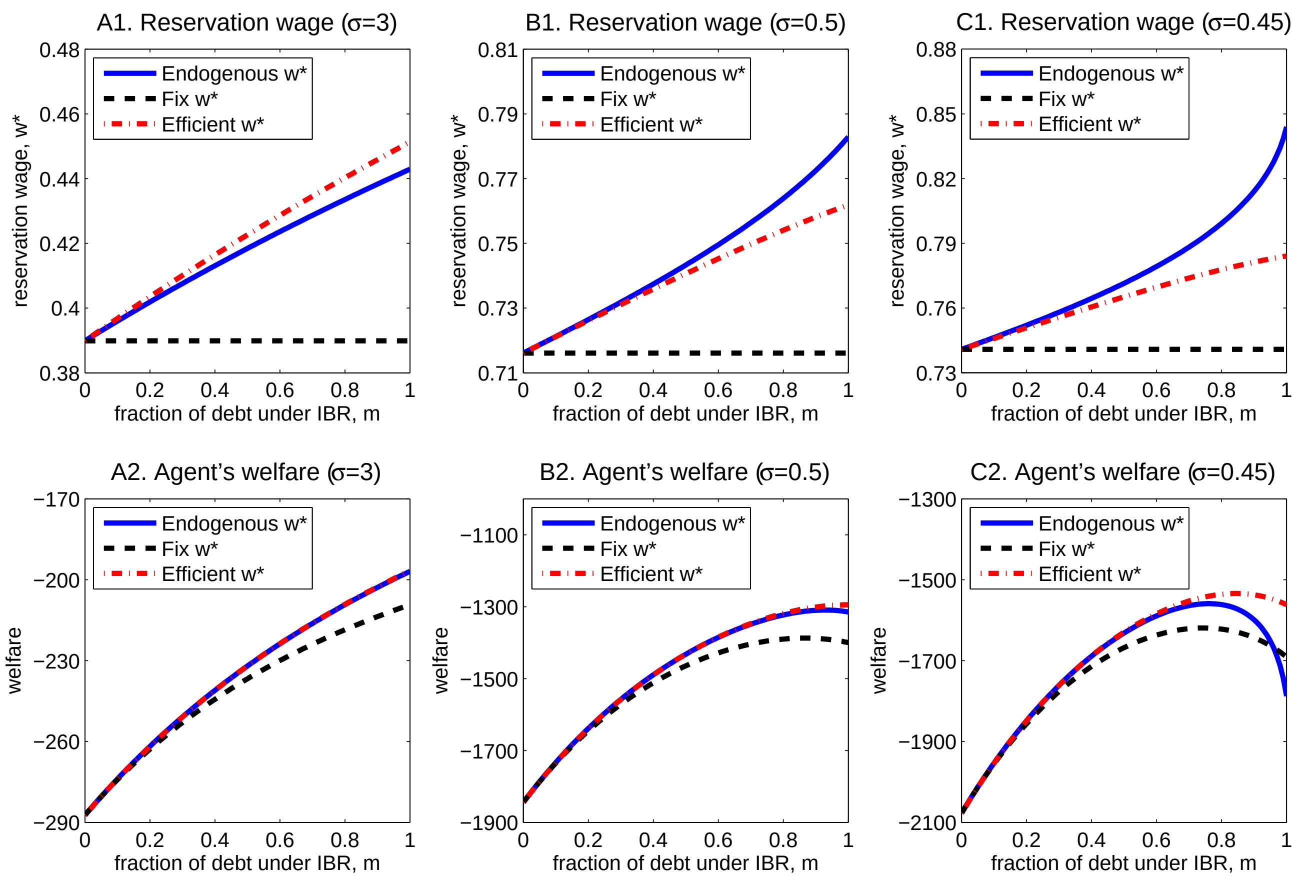
In panels A, I set $\sigma = 3$ to consider an empirically reasonable elasticity of labor supply, 0.33, according to Keane (2011). Panel A2 shows that welfare increases when a larger fraction of debt is made income contingent. It is clear that the inefficiency due to reservation wages is minimal as the welfare with endogenous reservation wages (blue solid line) is almost on top of that under the efficient contract (red dash-dotted line). Importantly, allowing the reservation wage to respond increases the agent's welfare relative to fixing the reservation wage at the beginning (black dashed line). This is because the reservation wage under the fixed repayment contract is too low compared to the efficient reservation wage. Increasing the fraction of income contingency raises the reservation wage, closing the gap to the efficient one (see panel A1) and lowering the repayment ratio.

In panels B, I dramatically increase the elasticity of labor supply to 2 by setting $\sigma = 0.5$. Similar to the result of Figure D.2, welfare first increases and then decreases due to the increasing distortion of income contingency on labor supply (see panel B2). The welfare with endogenous reservation wages is still higher than that with fixed reservation wages, but by contrast, the endogenous reservation wage is above the efficient one (see panel B1).

In panels C, I further increase the elasticity of labor supply to 2.22 by setting $\sigma = 0.45$. I obtain the result in which the debt-overhang channel dominates, and increasing the fraction of income contingency indirectly reduces welfare by increasing the reservation wage. Panel C2 shows that the agent's welfare would be higher if the reservation wage is fixed at the beginning. As shown in panel C1, this is essentially caused by the sharp increase in the reservation wage relative to the efficient one when a larger fraction of debt is made income contingent.

In sum, IBR increases the agent's welfare by directly providing insurance. The insurance leads to a higher reservation wage, which may or may not increase the agent's welfare. The key parameters governing whether a higher reservation wage is beneficial are the degree of risk aversion and the elasticity of labor supply. All else equal, a more risk-averse agent sets a lower reservation wage relative to the efficient one under the fixed repayment contract. Thus increasing the reservation wage by providing insurance increases welfare. A larger elasticity of labor supply intensifies the debt overhang channel.

[13] For example, in the extreme case with $\sigma = 0$, the second term in equation (D.52) vanishes to zero, and thus $w^*_{IBR} = \overline{w} > w^*_{EFI}$.




Note: This figure illustrates the reservation wage effect for different elasticities of labor supply. In panel A1, B1, and C1, the blue solid line plots the agent's reservation wage when the reservation wage is allowed to increase as a larger fraction of debt is made income contingent. The black dashed line plots the agent's reservation wage when the reservation wage is fixed at the initial value under the pure fixed repayment contract. The red dash-dotted line plots the agent's efficient reservation wage. The corresponding welfare is plotted in panel A2, B2, and C2. The elasticity of labor supply is 0.33 ($\sigma = 3$), 2 ($\sigma = 0.5$), and 2.22 ($\sigma = 0.45$) in panels A, B, and C. The figure is plotted using the GHH utility, $u(c, l) = [c - \phi l^{1+\sigma}/(1+\sigma)]^{1-\gamma}/(1-\gamma)$ and the beta distribution of wage offers, $Beta(a, b)$, with parameter values: $a = 2$, $b = 4$, $\gamma = 3$, $\theta = 0.1$, $\overline{w} = 1.1$, $\beta = 0.96$, $S = 1$, $\phi = 1$.

Figure D.3: A numerical illustration of the reservation wage effect for different elasticities of labor supply.

Thus when the incentive to work is distorted by IBR, it is more likely to result in a reservation wage too high compared to the efficient one. In this case, by committing to a lower reservation wage, the agent could increase her expected repayment, inducing the lender to set a lower repayment ratio, which consequently increases welfare. However, such commitment is not incentive compatible because ex-post a lower repayment ratio generates a steeper wage offer distribution due to the elastic labor supply. This motivates the agent to stay unemployed longer by setting a higher reservation wage, and the lender would take a loss on debt collection.

Despite the theoretical possibility, in reality, it is plausible that IBR indirectly increases welfare by increasing the reservation wage. This is due to two reasons. First, as suggested by the numerical examples in Figure D.3, a higher reservation wage reduces welfare only when the elasticity of labor supply is about two, while a consensus empirical estimate is usually below one. Second, the theoretical possibility roots from the inefficiency in IBR, which is designed to allow the lender to collect all debt in expectation. In other words, if the repayment ratio is fixed, instead of being varied with the endogenous reservation wage, then the inefficiency would disappear by construction. This is the case in reality, as the government is willing to take a loss by offering debt forgiveness for federal student loans.

## D.5 Optimal Repayment Contract

In theory, IBR is not the most efficient way to provide insurance because the repayment ratio is constant regardless of the level of income. In this subsection, I characterize the optimal repayment contract under the assumption that the reservation wage is not contractible. I show that the existence of search risks sets up the optimal contract that also considers the level of reservation wages. The implication is that the lender should provide more insurance in an economy with search risks, because this would increase the reservation wage. Therefore, IBR although not constrained efficient, is designed in the spirit of the optimal repayment contract as it both provides insurance and increases the reservation wage.

To gain some insight, let us begin with the first-best contract. The first-best contract not only provides full insurance against search risks but also sets the reservation wage to $\hat{w}$ to maximize expected income. When labor supply is inelastic, the first-best contract is also incentive compatible because perfect insurance makes the agent indifferent about the reservation wage. This suggests that in contrast to a model without search risks, insurance is more desirable in my model because income risks are controlled by the agent's endogenous job search decisions. The full insurance provided by the first-best contract not only directly increases welfare through consumption smoothing; but also indirectly increases welfare by making a higher reservation wage incentive compatible.

When labor supply is elastic, the first-best contract is not incentive compatible because supplying labor generates disutility. The second-best contract solves the problem in which the lender chooses a nonlinear repayment schedule $\alpha(z)$ conditional on earnings $z = wl$ subject to the recoverability constraint and the agent's incentive compatibility constraints on labor supply and the reservation wage. This problem is more complicated compared to the optimal income taxation problem solved by Mirrlees (1971) as there is an additional incentive compatibility constraint on the reservation wage.

Below I first show that when there is no job search (i.e., the reservation wage is fixed at $w^* = 0$), the mathematical problem is exactly the same as Mirrlees (1971)'s problem with a utilitarian social welfare function. I then show that my problem is different due to the introduction of endogenous job search decisions. I formulate the optimal contracting problem and use the perturbation approach inspired by Saez (2001) to elucidate the economic channels.

### D.5.1 Without Job Search

When the reservation wage $w^*$ is set to be 0, the agent accepts all wage offers drawn from $F(w)$ in the first period. Therefore, the agent's life-time utility conditional on receiving a wage offer $w$ is

$$W(w) = \frac{u(w, l)}{1 - \beta}, \tag{D.54}$$

where $l$ is the labor supply that satisfies the first-order condition.

To maximize the agent's expected life-time utility, the lender chooses an optimal nonlinear repayment schedule $\alpha^{SB}(z)$, as a function of the agent's earnings $z = wl$ to collect debt $S/\beta$. The nonlinear repayment schedule is not written on wage rates because wage rates are not observable or contractible.[14] The intercept $\alpha^{SB}(0)$ can be thought of as a lump-sum repayment or subsidy that is applied to any realization of earnings. The marginal repayment rate is $\alpha^{SB}(z)'$.

This problem is exactly the same as Mirrlees (1971) if we interpret it in the following way. There is a continuum of agents with different skills $w$ and homogeneous utility functions $\frac{u(c,l)}{1-\beta}$. They work in a static economy and optimally choose their labor supply $l$ in the tax system. The government values a utilitarian social welfare function and optimally designs a nonlinear tax schedule $\alpha^{SB}(z)$ in terms of earnings $z$ to maximize social welfare conditional on collecting $S/\beta$ revenue.

The problem is solved by Mirrlees (1971) by applying an optimal control approach on direct truth-telling mechanisms. The advantage of this approach comes from its rigorousness to obtain the technical conditions.[15] However, the derived formula is not useful to elucidate the economic intuitions underlying the optimal contract.

### D.5.2 With Job Search

Now I consider the optimal contracting problem with endogenous job search decisions as specified in section 3. The only departure from the problem of Mirrlees (1971) is that the agent chooses a reservation wage below which the wage offer is rejected. Therefore, in this problem, the types of agents in the problem of Mirrlees (1971) are restricted to a mass point with earnings $\theta$ with probability $F(w^*)$ and a continuum of types in $[w^*, \overline{w}]$ with density $\frac{f(w)}{1-F(w^*)}$, where $w^*$ is chosen by the agent to maximize her welfare.

Facing any nonlinear repayment contract $\alpha(z)$ in terms of earnings $z$, the agent makes two decisions to maximize her welfare. First, the agent chooses a reservation wage $w^*$. Second, conditional on accepting the wage offer $w$, the agent chooses her labor supply $l$. Therefore, the resulting distribution of earnings $H(z)$ depends both on the exogenous wage offer distribution $F(w)$ and the repayment schedule $\alpha(z)$.

Below, I use a perturbation approach inspired by Saez (2001) to characterize the shape of the optimal repayment contract $\alpha^{SB}(z)$. For tractability, I make the following assumptions.

**Assumption 1.** *Earnings $z$ and utility $u(z - \alpha^{SB}(z), l^{SB}(z))$ weakly increase with wage rates $w$ under the optimal repayment contract $\alpha^{SB}(z)$.*

**Assumption 2.** *The optimal repayment contract $\alpha^{SB}(z)$ is twice differentiable for all $z$.*

Assumption 1 is saying that the agent earns more and enjoys higher welfare at jobs with higher wage rates. This is intuitively reasonable given that the monotonicity condition in the mechanism

[14] If wage rates are contractible, then the first-best allocation is attainable because labor supply would not be distorted by repayment contracts. It is reasonable to assume that wage rates are unobservable because if they are observable, then labor supply is also observable from wage income. But this contradicts with the assumption made in the optimal income taxation literature.

[15] For example, in order to have the local incentive-compatibility constraint being sufficient, the problem is required to satisfy the Spence-Mirrlees single crossing condition and the monotonicity condition.

design problem of Mirrlees (1971) requires net earnings $z - \alpha^{SB}(z)$ to be weakly increasing in $w$. This assumption ensures that there is an injective function under the optimal contract $\alpha^{SB}(z)$, $w \mapsto z = q(w)$. Thus I denote $z^*$ as the earnings corresponding to the reservation wage offer $w^*$, i.e., $z^* = q(w^*)$.

Assumption 2 comes from Saez (2001). This assumption has additional meaning in the problem I solve because it also restricts the specification of contract off the equilibrium, i.e., for $z \in (\theta, z^*)$. In general, because the agent rejects the wage offer whenever the resulting earnings are below $z^*$, there exist infinite numbers of optimal repayment contracts in my problem, and some of them could have a discontinuous jump at $z^*$.[16] This assumption ensures that the reservation wage is derived from a first-order condition instead of being a corner solution. That is, when the reservation wage is slightly changed, the change in the agent's welfare is of second order.

Denote $\lambda < 0$ as the Lagrangian multiplier associated with the lender's recoverability constraint,

$$\frac{H(z^*)}{1-\beta H(z^*)}\alpha^{SB}(\theta) + \frac{1}{(1-\beta)[1-\beta H(z^*)]}\int_{z^*}^{\infty} \alpha^{SB}(z)dH(z) = \frac{S}{\beta}. \tag{D.55}$$

The multiplier $\lambda$ is also the shadow value measuring the change in the agent's welfare when the amount of debt marginally increases.[17] Denote $g(z) > 0$ as the marginal value of consumption for the agent with earnings $z$ under the optimal repayment contract, expressed in terms of the shadow cost of debt $(-\lambda)$, i.e.,

$$g(z) = \frac{u_1(z - \alpha^{SB}(z), l^{SB}(z))}{-\lambda}, \tag{D.56}$$

where $l^{SB}(z)$ corresponds to the labor supply at earnings $z$ under the optimal contract $\alpha^{SB}(z)$.

I follow Saez (2001) and consider a small perturbation around the optimal repayment schedule $\alpha^{SB}(z)$. Suppose that the marginal repayment rate is increased by $d\alpha$ for earnings between $z$ and $z + dz$, where $z \geq z^*$. This would generate the following effects on expected repayment $R$, defined as:

$$R = \frac{H(z^*)}{1-\beta H(z^*)}\alpha^{SB}(\theta) + \frac{1}{(1-\beta)[1-\beta H(z^*)]}\int_{z^*}^{\infty} \alpha^{SB}(z)dH(z). \tag{D.57}$$

**Mechanical effect** The agent pays $d\alpha dz$ more when her earnings are above $z$, with probability $1 - H(z)$. Thus expected repayment increases by

$$M = \frac{1-H(z)}{(1-\beta)[1-\beta H(z^*)]}d\alpha dz. \tag{D.58}$$

**Elasticity effect** The increase in the marginal repayment rate distorts labor supply when the agent's earnings are between $z$ and $z + dz$, which consequently affects expected repayment. The change in earnings is caused by two effects. First, there is a direct effect due to the increase in $d\alpha$. Second, there is

[16] For example, given the optimal contract $\alpha^{SB}(z)$. We can specify $\tilde{\alpha}^{SB}(z)$ such that $\tilde{\alpha}^{SB}(z) = \alpha^{SB}(z)$ $for\ z \geq z^*$ and $z - \tilde{\alpha}^{SB}(z) = \theta - \alpha^{SB}(\theta)$ $for\ z < z^*$. Under $\tilde{\alpha}^{SB}(z)$, the net earnings are flat up to the reservation earnings $z^*$, and there is a discontinuous jump in net earnings at $z^*$. The contract $\tilde{\alpha}^{SB}(z)$ is incentive compatible because the agent has no incentive to change her reservation earnings $z^*$ as reducing this lowers her utility more than what would be under $\alpha^{SB}(z)$. Moreover, $\tilde{\alpha}^{SB}(z)$ also satisfies the lender's recoverability constraint so it is an optimal contract.

[17] The negative of $\lambda$ corresponds to the social value of public funds defined by Saez (2001).

an indirect effect as the agent would face a different marginal repayment rate when her earnings are changed by the direct effect.

As noted by Saez (2001), the direct effect can be decomposed into two parts: an overall uncompensated increase in the marginal rate and an overall increase in virtual income. Therefore, the relevant one that determines the behavioral response is the Hicksian (compensated) elasticity of earnings, which is defined as

$$\zeta^c(z) = \frac{1-\alpha^{SB}(z)'}{z} \frac{\partial z}{\partial(1-\alpha^{SB}(z)')}\bigg|_u. \tag{D.59}$$

Suppose that the two effects result in an earnings change by $\Delta$, then the direct effect is $-\zeta^c(z)z\frac{d\alpha}{1-\alpha^{SB}(z)'}$, and the indirect effect is $-\zeta^c(z)z\frac{\Delta\alpha^{SB}(z)''}{1-\alpha^{SB}(z)'}$. Hence,

$$\Delta = -\zeta^c(z)z\frac{d\alpha}{1-\alpha^{SB}(z)'} - \zeta^c(z)z\frac{\Delta\alpha^{SB}(z)''}{1-\alpha^{SB}(z)'}. \tag{D.60}$$

This implies that

$$\Delta = -\zeta^c(z)z\frac{d\alpha}{1-\alpha^{SB}(z)'+\zeta^c(z)z\alpha^{SB}(z)''}. \tag{D.61}$$

Following Saez (2001), I assume that $1-\alpha^{SB}(z)'+\zeta^c(z)z\alpha^{SB}(z)'' > 0$ so that bunching of types does not occur. The elasticity effect on expected repayment is

$$\begin{aligned} E &= \frac{\Delta\alpha^{SB}(z)'h(z)dz}{(1-\beta)[1-\beta H(z^*)]} \\ &= -\frac{\zeta^c(z)z\alpha^{SB}(z)'}{1-\alpha^{SB}(z)'+\zeta^c(z)z\alpha^{SB}(z)''}\frac{h(z)}{(1-\beta)[1-\beta H(z^*)]}d\alpha dz. \end{aligned} \tag{D.62}$$

**Income effect** If the agent accepts a wage offer generating earnings above $z+dz$, her earnings are reduced by $d\alpha dz$ due to the higher marginal rate between $z$ and $z+dz$. This would generate an income effect that induces the agent to work more. As a result, for any $x > z+dz$, earnings increase by $\Delta(x)$, which in turn increases expected repayment. The earnings response $\Delta(x)$ is due to two effects. First, there is a direct effect due to the increase in marginal rate $d\alpha$ between $z$ and $z+dz$. Second, there is an indirect effect due to the change in marginal rates caused by the shift in earnings.

Let $\eta(z) \le 0$ denote the income effect and $\zeta^u(z)$ denote the Marshallian (uncompensated) elasticity of earnings at earnings $z$, thus the income effect is derived by the Slutsky equation,

$$\zeta^u(z) = \frac{1-\alpha^{SB}(z)'}{z}\frac{\partial z}{\partial(1-\alpha^{SB}(z)')}; \tag{D.63}$$

$$\eta(z) = \zeta^u(z) - \zeta^c(z). \tag{D.64}$$

Therefore, the direct effect is $-\frac{\eta(x)d\alpha dz}{1-\alpha^{SB}(x)'}$ and the indirect effect is $-\zeta^c(x)x\frac{\alpha^{SB}(x)''\Delta(x)}{1-\alpha^{SB}(x)'}$, and the change in earnings is

$$\Delta(x) = -\frac{\eta(x)d\alpha dz}{1-\alpha^{SB}(x)'} - \zeta^c(x)x\frac{\alpha^{SB}(x)''\Delta(x)}{1-\alpha^{SB}(x)'}, \tag{D.65}$$

which implies

$$\Delta(x) = -\eta(x)\frac{d\alpha dz}{1-\alpha^{SB}(x)' + x\zeta^c(x)\alpha^{SB}(x)''}. \tag{D.66}$$

The total income effect on expected repayment is

$$I = -\frac{d\alpha dz}{(1-\beta)[1-\beta H(z^*)]}\int_z^\infty \eta(x)\frac{\alpha^{SB}(x)'}{1-\alpha^{SB}(x)' + x\zeta^c(x)\alpha^{SB}(x)''}h(x)dx. \tag{D.67}$$

**Reservation wage effect** There is a fourth effect on expected repayment due to the change in reservation earnings, which is not in the problem of Mirrlees (1971). The reservation earnings are determined by the following indifference equation:

$$\frac{u(z^* - \alpha^{SB}(z^*), l^{SB}(z^*))}{1-\beta} = \frac{u(\theta - \alpha^{SB}(\theta), 0)}{1-\beta H(z^*)} + \frac{\beta}{(1-\beta)[1-\beta H(z^*)]}\int_{z^*}^\infty u(x - \alpha^{SB}(x), l^{SB}(x))dH(x), \tag{D.68}$$

where the LHS of this equation represents the value of being employed at the reservation earnings $z^*$, and the RHS represents the value of staying unemployed. Assumption 2 ensures that the reservation earnings also satisfy the first-order condition. Rearranging it:

$$1 = \frac{\beta}{1-\beta}\int_{z^*}^\infty \frac{u(x-\alpha^{SB}(x), l^{SB}(x)) - u(z^* - \alpha^{SB}(z^*), l^{SB}(z^*))}{u(z^* - \alpha^{SB}(z^*), l^{SB}(z^*)) - u(\theta - \alpha^{SB}(\theta), 0)}dH(x). \tag{D.69}$$

Assumption 1 ensures that the integrand is non-negative and decreasing in $z^*$. The integration is executed from $z^*$ to infinity, thus the RHS of equation (D.69) decreases with $z^*$. The increase in the marginal repayment rate $d\alpha$ between $z$ and $z + dz$ reduces $u(x - \alpha^{SB}(x), l^{SB}(x))$ for all $x > z$, thus lowering the RHS of equation (D.69). This implies that the reservation earnings $z^*$ would decrease.

For $x > z$, the change $d\alpha$ would change $u(x - \alpha^{SB}(x), l^{SB}(x))$ by

$$\begin{aligned} du(x) &= -u_1(x - \alpha^{SB}(x), l^{SB}(x))d\alpha dz \\ &= g(x)\lambda d\alpha dz. \end{aligned} \tag{D.70}$$

Note that the elasticity effect and the income effect discussed above indicate that labor supply $l^{SB}(x)$ would also change due to the change $d\alpha$, but the Envelope Theorem implies that such a change does not have a first-order effect on utility. Differentiating equation (D.68) and substituting (D.70), we obtain

$$dz^* = d\alpha dz \frac{\beta\lambda}{[1-\beta H(z^*)]u_z(z^*)}\int_z^\infty g(x)dH(x), \tag{D.71}$$

where $u_z(z) = \frac{du(z-\alpha^{SB}(z), l^{SB}(z))}{dz}$ denotes the marginal change in utility due to a marginal change in earnings at $z$ under the optimal contract $\alpha^{SB}(z)$.

The change in reservation earnings $dz^*$ does not affect the agent's welfare due to the envelope condition from Assumption 2. However, it affects expected repayment $R$ determined by equation (D.57).

Define $\zeta^{z^*}$ as the elasticity of expected repayment with respect to the reservation earnings,

$$\zeta^{z^*} = \frac{\partial R/R}{\partial z^*/z^*}. \tag{D.72}$$

Differentiating (D.57), we obtain

$$\begin{aligned}\zeta^{z^*} &= \frac{\beta R + \alpha^{SB}(\theta) - \dfrac{\alpha^{SB}(z^*)}{1-\beta}}{[1-\beta H(z^*)]R} z^* h(z^*) \\ &= \frac{\beta S + \beta\alpha^{SB}(\theta) - \dfrac{\beta\alpha^{SB}(z^*)}{1-\beta}}{[1-\beta H(z^*)]S} z^* h(z^*),\end{aligned} \tag{D.73}$$

where the second equation is obtained by substituting $R = S/\beta$.

In general, $\zeta^{z^*}$ could be positive or negative. The discussion in Appendix D.4 suggests that $\zeta^{z^*} > 0$ for empirically reasonable elasticities of labor supply. Therefore, higher reservation earnings increase expected repayment. Using equations (D.71) and (D.73), we obtain the reservation wage effect on expected repayment:

$$\begin{aligned}RW &= \frac{dz^*}{z^*}\zeta^{z^*} R \\ &= d\alpha dz \frac{S\lambda\zeta^{z^*}}{[1-\beta H(z^*)]u_z(z^*)z^*} \int_z^\infty g(x)dH(x).\end{aligned} \tag{D.74}$$

**Deriving the Optimal Contract During Employment** The small perturbation around the optimal contract should have no first-order effect on welfare. Therefore, the sum of the four effects, $M$, $E$, $I$, and $RW$, multiplied by the shadow cost of debt $(-\lambda)$ should be equal to the agent's expected welfare loss when earnings are above $z$. The agent's welfare under $\alpha^{SB}(z)$ is

$$\text{Welfare}_{SB} = \frac{H(z^*)u(\theta - \alpha^{SB}(\theta), 0)}{1-\beta H(z^*)} + \int_{z^*}^\infty \frac{u(x-\alpha^{SB}(x), l^{SB}(x))}{(1-\beta)[1-\beta H(z^*)]} dH(x) \tag{D.75}$$

The expected welfare loss is

$$\begin{aligned}WL &= d\alpha dz \int_z^\infty \frac{u_1(x-\alpha^{SB}(x), l^{SB}(x))}{(1-\beta)[1-\beta H(z^*)]} dH(x) \\ &= d\alpha dz \int_z^\infty \frac{-\lambda g(x)}{(1-\beta)[1-\beta H(z^*)]} dH(x).\end{aligned} \tag{D.76}$$

Again, the Envelope Theorem implies that the change in labor supply has a second-order effect on welfare. At the optimum,

$$WL = -\lambda(M + E + I + RW), \tag{D.77}$$

which implies

$$\underbrace{\int_z^\infty g(x)dH(x)}_{\text{direct welfare loss}} = \underbrace{1-H(z)}_{\text{mechanical effect}} \underbrace{-\frac{z\zeta^c(z)\alpha^{SB}(z)'}{1-\alpha^{SB}(z)'+z\zeta^c(z)\alpha^{SB}(z)''}h(z)}_{\text{elasticity effect}}$$
$$\underbrace{-\int_z^\infty \eta(x)\frac{\alpha^{SB}(x)'}{1-\alpha^{SB}(x)'+x\zeta^c(x)\alpha^{SB}(x)''}dH(x)}_{\text{income effect}}$$
$$+\underbrace{\frac{S(1-\beta)\lambda\zeta^{z^*}}{u_z(z^*)z^*}\int_z^\infty g(x)dH(x)}_{\text{reservation wage effect}}. \tag{D.78}$$

This equation implicitly determines the optimal contract $\alpha^{SB}(z)$. It is different from the one derived by Saez (2001) due to the existence of the reservation wage effect. As a result, it does not admit an explicit solution for $\alpha^{SB}(z)$ because the elasticity of earnings with respect to the reservation earnings, $\zeta^{z^*}$, is a function of $\alpha^{SB}(z)$.

To gain some intuitions, consider the case with inelastic labor supply, which implies that there is no elasticity effect or income effect in equation (D.78). If there are no endogenous search decisions, the reservation wage effect is also absent. Then the optimal contract requires $\int_z^\infty g(x)dH(x) = 1-H(z)$ for all $z > z^*$. This happens only when $g(z) = 1, \forall z > z^*$, suggesting perfect insurance against earnings risks.

When there are search risks, the direct welfare loss is equal to the sum of the mechanical effect and the reservation wage effect. If the agent is provided with perfect insurance, $g(z) = 1$, then the marginal utility does not change when different earnings offers are accepted. This implies that the term $u_z(z^*)$ in the reservation wage effect is equal to zero. In this case, for the reservation wage effect to be well defined, it is required that $\zeta^{z^*} = 0$, which happens when the reservation earnings $z^*$ is set to maximize expected repayment.

Note that the lender can set the reservation wage to maximize expected repayment precisely because the agent with inelastic labor supply is indifferent among different reservation wages when being perfectly insured. Hence, any reservation wage is incentive compatible. This simple discussion with inelastic labor supply highlights the role of reservation wages in optimal contract design: in the context of elastic labor supply, the optimal contract not only cares about the tradeoff between efficiency (incentive to work) and insurance, but also to some extent, uses the reservation wage to increase expected repayment in order to have a smaller distortion on efficiency.

Equation (D.78) characterizes the formula that implicitly determines the optimal marginal repayment rate during employment. In the following, I derive the optimal repayment during unemployment.

**Deriving the Optimal Contract During Unemployment** Suppose that repayment is increased by $d\alpha$ during unemployment, which is achieved by smoothly perturbing the repayment schedule below $z^*$ so that Assumption 2 is still satisfied. This is going to have a mechanical effect and a reservation wage effect on expected repayment.

The mechanical effect is given by

$$M = \frac{H(z^*)}{1 - \beta H(z^*)} d\alpha, \tag{D.79}$$

which captures the fact that the agent repays more during unemployment. Similar to equation (D.70), for earnings $\theta$, the increase in repayment reduces utility during unemployment by

$$du(\theta) = -u_1(\theta - \alpha^{SB}(\theta), 0) d\alpha = g(\theta) \lambda d\alpha. \tag{D.80}$$

The reservation earnings are determined by equation (D.68). Differentiating this equation and substituting (D.80) yields:

$$dz^* = d\alpha \frac{(1-\beta)\lambda g(\theta)}{[1 - \beta H(z^*)] u_z(z^*)}. \tag{D.81}$$

Thus the reservation wage effect is

$$\begin{aligned} RW &= \frac{dz^*}{z^*} \zeta^{z^*} R \\ &= d\alpha \frac{S(1-\beta)\lambda g(\theta) \zeta^{z^*}}{\beta [1 - \beta H(z^*)] u_z(z^*) z^*}. \end{aligned} \tag{D.82}$$

According to equation (D.75), this perturbation generates a direct welfare loss:

$$\begin{aligned} WL &= \frac{H(z^*)}{1 - \beta H(z^*)} u_1(\theta - \alpha^{SB}(\theta), 0) d\alpha \\ &= -\frac{H(z^*)}{1 - \beta H(z^*)} g(\theta) \lambda d\alpha. \end{aligned} \tag{D.83}$$

At the optimum,

$$WL = -\lambda (M + RW), \tag{D.84}$$

which yields

$$\underbrace{H(z^*) g(\theta)}_{\text{direct welfare loss}} = \underbrace{H(z^*)}_{\text{mechanical effect}} + \underbrace{\frac{S(1-\beta)\lambda \zeta^{z^*}}{\beta u_z(z^*) z^*} g(\theta)}_{\text{reservation wage effect}}. \tag{D.85}$$

If the reservation earnings are fixed, then the reservation wage effect is absent in equation (D.85). In this case, the optimal contract subsidizes unemployment such that $g(\theta) = 1$, i.e., to the point where the marginal utility of consumption during unemployment is equal to the shadow cost of debt. This is because there is no behavioral response during unemployment, thus it is always optimal to equalize the cost of fund to the marginal utility of consumption when the agent is unemployed. When there is a negative reservation wage effect, the optimal contract sets $g(\theta) < 1$, indicating that the lender subsidizes the agent more during unemployment. Intuitively, this is because providing more liquidity to unemployment incentivizes the agent to increase her reservation wage and search longer, which would raise expected repayment.

# E Quantitative Model Details

In this appendix section, I fill in the details for my quantitative model in section 4. I first present the value functions for non-defaulted agents and jobs matched with these agents. Then I illustrate the value functions under the fixed repayment plan and IBR. Finally, I illustrate the wage function with respect to student debt and show that the amount of student debt does not affect the wage rate much through Nash bargaining.

## E.1 Value Functions for Non-Defaulted Agents

### E.1.1 Unemployed Workers

An unemployed worker who has not defaulted yet has the option to default, thus her value is

$$U(\Omega_t) = \max\left\{ \underbrace{\hat{U}(\Omega_t)}_{\text{value of default}}, \underbrace{\tilde{U}(\Omega_t)}_{\text{value of non-default}} \right\}, \tag{E.1}$$

where $\hat{U}(\Omega_t)$ and $\tilde{U}(\Omega_t)$ represent the value of default and not default for unemployed workers:

$$\begin{aligned} \hat{U}(\Omega_t) = \max_{c_t, l_t} \quad & u(c_t, l_t) \underbrace{-\eta}_{\text{default cost}} + \beta\Bigg[ \underbrace{\lambda^u \int_{x \geq \rho_u^d} W^d(\Omega_{t+1}, x, \rho_u^d) dV(x)}_{\text{accept the job}} + \underbrace{[1 - \lambda^u + \lambda^u V(\rho_u^d)] U^d(\Omega_{t+1})}_{\text{not accept the job}} \Bigg] \\ \text{subject to} \quad & b_{t+1} = (1+r)b_t + \varkappa\theta^{1-\tau} - c_t, \\ & s_{t+1} = (1+r^s)s_t, \\ & c_t \geq \underline{c}, \\ & b_{t+1} \geq 0, \end{aligned} \tag{E.2}$$

$$\begin{aligned} \tilde{U}(\Omega_t) = \max_{c_t, l_t} \quad & u(c_t, l_t) + \beta\Bigg[ \underbrace{\lambda^u \int_{x \geq \rho_u} W(\Omega_{t+1}, x, \rho_u) dV(x)}_{\text{accept the job}} + \underbrace{[1 - \lambda^u + \lambda^u V(\rho_u)] U(\Omega_{t+1})}_{\text{not accept the job}} \Bigg] \\ \text{subject to} \quad & b_{t+1} = (1+r)(b_t - y_t^{fix}) + \varkappa\theta^{1-\tau} - c_t, \\ & s_{t+1} = (1+r^s)(s_t - y_t^{fix}), \\ & c_t \geq \underline{c}, \\ & b_{t+1} \geq 0, \end{aligned} \tag{E.3}$$

The difference between the recursive formula of $\hat{U}(\Omega_t)$ and $\tilde{U}(\Omega_t)$ lies in the default cost and debt repayment. The agent incurs a default cost $\eta$ if she chooses to default in period $t$, and the benefit of this is that no payment is elicited in that period.

Denote $d(\Omega_t)$ as the default decision for unemployed workers,

$$d(\Omega_t) = \begin{cases} 1, & \text{if } \hat{U}(\Omega_t) > \tilde{U}(\Omega_t) \\ 0, & \text{if } \hat{U}(\Omega_t) \leq \tilde{U}(\Omega_t) \end{cases} \tag{E.4}$$

### E.1.2 Employed Workers

If the agent has not defaulted, her value function is:

$$W(\Omega_t, \rho, \rho') = \max \Big\{ \underbrace{\hat{W}(\Omega_t, \rho, \rho')}_{\text{value of default}}, \underbrace{\tilde{W}(\Omega_t, \rho, \rho')}_{\text{value of non-default}} \Big\}. \tag{E.5}$$

where $\hat{W}(\Omega_t, \rho, \rho')$ and $\tilde{W}(\Omega_t, \rho, \rho')$ represent the value of default and not default for employed workers:

$$\begin{aligned} \hat{W}(\Omega_t, \rho, \rho') = \max_{c_t, l_t} \quad & u(c_t, l_t) \underbrace{- \eta}_{\text{default cost}} + \beta \Big\{ \underbrace{\kappa U^d(\Omega_{t+1})}_{\text{job separation}} + \underbrace{(1-\kappa) \Big[ [1 - \lambda^e + \lambda^e V(\rho')] W^d(\Omega_{t+1}, \rho, \rho')}_{\text{not poached or poached by a low vacancy}} \\ & + \lambda^e \Big( \underbrace{\int_{x \geq \rho} W^d(\Omega_{t+1}, x, \rho) dV(x)}_{\text{transition to a new vacancy}} + \underbrace{\int_{\rho' < x < \rho} W^d(\Omega_{t+1}, \rho, x) dV(x)}_{\text{negotiation for a wage rise}} \Big) \Big] \Big\}, \\ \text{subject to} \quad & b_{t+1} = (1+r) b_t + \varkappa [w^e(\Omega_t, \rho, \rho') l_t]^{1-\tau} - c_t, \\ & s_{t+1} = (1 + r^s) s_t. \\ & c_t \geq \underline{c}, \\ & b_{t+1} \geq 0. \end{aligned} \tag{E.6}$$

$$\begin{aligned} \tilde{W}(\Omega_t, \rho, \rho') = \max_{c_t, l_t} \quad & u(c_t, l_t) + \beta \Big\{ \underbrace{\kappa U(\Omega_{t+1})}_{\text{job separation}} + \underbrace{(1-\kappa) \Big[ [1 - \lambda^e + \lambda^e V(\rho')] W(\Omega_{t+1}, \rho, \rho')}_{\text{not poached or poached by a low vacancy}} \\ & + \lambda_e \Big( \underbrace{\int_{x \geq \rho} W(\Omega_{t+1}, x, \rho) dV(x)}_{\text{transition to a new vacancy}} + \underbrace{\int_{\rho' < x < \rho} W(\Omega_{t+1}, \rho, x) dV(x)}_{\text{negotiation for a wage rise}} \Big) \Big] \Big\} \\ \text{subject to} \quad & b_{t+1} = (1+r)(b_t - y_t^{fix}) + \varkappa [w^e(\Omega_t, \rho, \rho') l_t]^{1-\tau} - c_t, \\ & s_{t+1} = (1 + r^s)(s_t - y_t^{fix}), \\ & c_t \geq \underline{c}, \\ & b_{t+1} \geq 0. \end{aligned} \tag{E.7}$$

Denote $d(\Omega_t, \rho, \rho')$ as the default decision for employed workers,

$$d(\Omega_t, \rho, \rho') = \begin{cases} 1, & \text{if } \hat{W}(\Omega_t, \rho, \rho') > \tilde{W}(\Omega_t, \rho, \rho'), \\ 0, & \text{if } \hat{W}(\Omega_t, \rho, \rho') \leq \tilde{W}(\Omega_t, \rho, \rho'). \end{cases} \tag{E.8}$$

## E.2 Value Functions for Jobs Filled with Non-Defaulted Agents

The value of a job filled by a worker who has not defaulted yet is

$$\begin{aligned} J(\Omega_t, \rho, \rho') &= \underbrace{[Az_t\rho - w^e(\Omega_t, \rho, \rho')]l(\Omega_t, \rho, \rho')}_{\text{production profit in current period}} \\ &+\beta(1-\kappa)\Bigg\{ \underbrace{d(\Omega_t, \rho, \rho')}_{\text{default}} \Bigg[ \underbrace{\lambda^e \int_{\rho'<x<\rho} J^d(\Omega_{t+1}, \rho, x) dV(x)}_{\text{negotiation for a wage rise}} + \underbrace{[1-\lambda^e+\lambda^e V(\rho')] J^d(\Omega_{t+1}, \rho, \rho')}_{\text{not poached or poached by a low vacancy}} \Bigg] \\ &+ \underbrace{(1-d(\Omega_t, \rho, \rho'))}_{\text{not default}} \Bigg[ \underbrace{\lambda^e \int_{\rho'<x<\rho} J(\Omega_{t+1}, \rho, x) dV(x)}_{\text{negotiation for a wage rise}} + \underbrace{[1-\lambda^e+\lambda^e V(\rho')] J(\Omega_{t+1}, \rho, \rho')}_{\text{not poached or poached by a low vacancy}} \Bigg] \Bigg\}. \end{aligned}$$

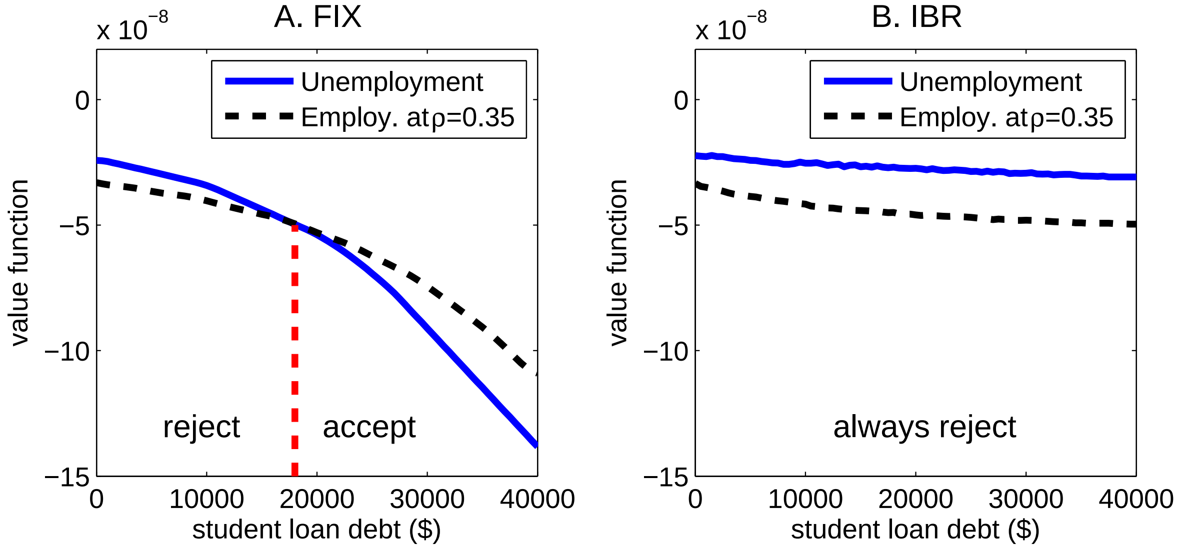
The match surplus relative to unemployment is given by

$$Surplus(\Omega_t, \rho, \rho') = W(\Omega_t, \rho, \rho') - U(\Omega_t) + J(\Omega_t, \rho, \rho'). \tag{E.9}$$

## E.3 Illustration of Value Functions

In this subsection, I illustrate the underlying mechanism of IBR by plotting the value functions. In panel A of Figure E.4, I plot the value function under the fixed repayment plan for an unemployed agent and the value function that could be achieved if the agent accepts a job with productivity $\rho = 0.35$ and negotiation benchmark $\rho' = 0.35$. Panel A illustrates the key mechanism of student loan debt by showing that the unemployed value function decreases faster with debt compared to the employed value function. As a result, there is an intersection between the two curves. In this example, when the level of student loan debt is below $18,000, the agent rejects the job offer and stays unemployed. When the level of debt is above $18,000, the agent takes the job. My theoretical analysis has revealed the mechanism for this observation: when the agent is burdened with more debt, she not only becomes more risk averse but also has larger liquidity needs, both increasing the marginal utility of consumption and pushing her toward accepting the job offer.

Panel B plots the value functions under IBR. It shows that under IBR, a higher level of debt reduces the value only slightly for both unemployed agents and employed agents. This is because there is much better risk sharing provided by IBR. First, IBR allows the agent to repay less when she has lower income, especially during unemployment. Second, there is debt forgiveness after 25 years, which convexifies the value functions. In this particular example, there is a sharp comparison as the unemployed agent always rejects the job offer with productivity ($\rho = 0.35$), and continues job search.



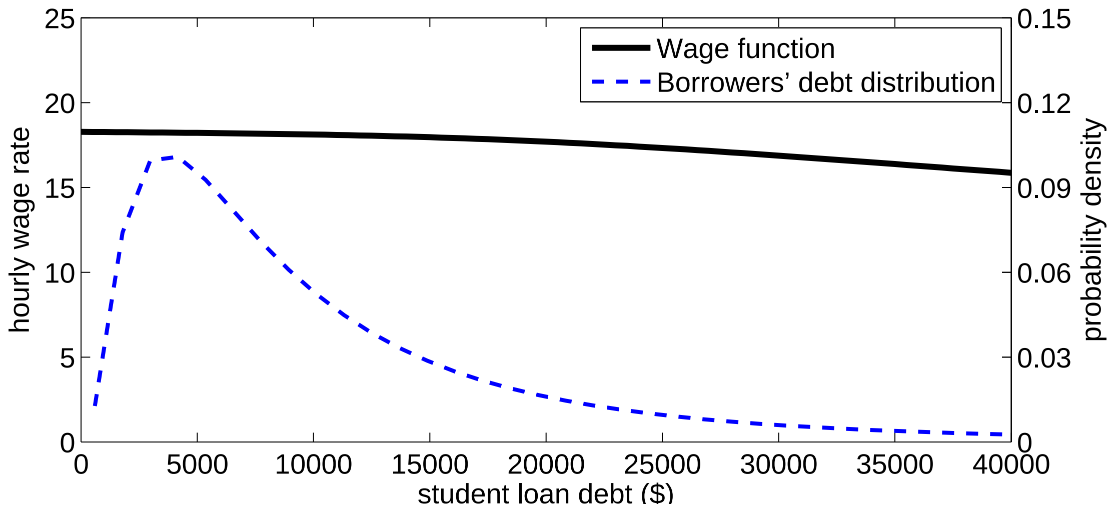

Note: This figure plots the value function with respect to the level of student loan debt for the agent with zero wealth in the first year after college graduation ($t = 1$). The left panel plots the value functions under the fixed repayment plan. The solid line represents the value function of an unemployed agent and the dashed line represents the value function of being employed at a job with productivity $\rho = 0.35$ and negotiation benchmark $\rho' = 0.35$. The right panel plots the value functions under IBR. In this example, when the level of student loan debt is below $18,000, the agent rejects the job offer and stays unemployed if she is under the fixed repayment plan. However, when the level of debt is above $18,000, the agent takes the job. This is in contrast to IBR, under which the agent always rejects the job offer.

Figure E.4: An illustration of the value functions under the fixed repayment plan and IBR.

## E.4 Wage Function

The wage rate is renegotiated in every period, reflecting the change in $\Omega$. The assumption of Nash bargaining links workers' wage rates to their characteristics, implying that wealth, student loan debt, and labor productivity can influence income. As argued by Krusell, Mukoyama and Sahin (2010), it is logical to assume that workers have the incentive to bargain for higher wages if outside options are strong. Moreover, the results under Nash bargaining are useful for comparison with the existing literature, because it is the most commonly used assumption under risk neutrality.

One concern of applying Nash bargaining to model wage determination is that the change in student loan debt could change the wage rate that maximizes the bargaining problem (4.13). This confounds the mechanism I hope to quantify, which is how student loan debt affects wage income by affecting job search decisions. As shown in Figure E.5, the wage rate derived from Nash bargaining is not very responsive to the level of debt. This is due to the existence of two countervailing forces in problem (4.13). On the one hand, a larger debt repayment reduces the value of the outside option $U(\Omega)$ more than the reduction in $W(\Omega, \rho, w)$ because the marginal value of liquidity is higher during unemployment when income is relatively lower. This increases worker's surplus from the match, $W(\Omega, \rho, w) - U(\Omega)$, reducing the wage rate for the worker. On the other hand, a larger debt repayment increases the marginal value of liquidity for the worker at the current job due to the reduction in consumption. This increases the sensitivity of the worker's employment value with respect to the wage rate, $\partial W(\Omega, \rho, w)/\partial w$, increasing the wage rate for the worker. The impact of the bargaining channel could be large when the level of student loan debt is very high, which is not the case in my estimation sample. This result is also consistent with Krusell, Mukoyama and Sahin (2010)'s finding that wage differentials created by the heterogeneity of asset and Nash bargaining are small.




This figure plots the wage function for a typical agent in the first year after college graduation for different amounts of student loan debt. I consider the agent having average wealth ($4,500) and being employed at a job with average productivity (0.725) and with the negotiation benchmark's productivity being set at the reservation productivity (0.325). It shows that increasing the amount of student loan debt from $0 to the average amount ($19,000) reduces the wage rate by about 1.0% (from $18.26 to $18). For agents with other job productivity and negotiation benchmark, the sensitivity of wage rates with respect to student loan debt is similar.

Figure E.5: An illustration of the wage function under the fixed repayment plan.

In the figure, I consider the agent having average wealth ($4,500) and being employed at a job with average productivity (0.725) and with the negotiation benchmark's productivity being set at the reservation productivity (0.325). It shows that increasing the amount of student loan debt from $0 to the average amount ($19,000) reduces the wage rate by about 1.0% (from $18.26 to $18). For agents with other job productivity and negotiation benchmark, the sensitivity of wage rates with respect to student loan debt is similar. This suggests that the bargaining channel confounds the mechanism but quantitatively it is much less important. Specifically, as shown in Table 6, normalized borrowers' wage income is 4.2% lower compared to that of non-borrowers, suggesting that roughly three quarters of the reduction in wage income is caused by the mechanism that reduces the reservation wage, and one quarter is caused by the nash bargaining channel which reflects the change in outside options.

Figure E.5 also indicates that the wage rate is more sensitive to student loan debt when the amount of debt is very high. When student loan debt increases from $0 to $40,000, the reduction in wage rates caused by the nash bargaining channel alone is as large as 13.1%. However, these rare cases are not driving the quantitative results of my model, because most students have loan amounts below $20,000 according to the estimated distribution.

Finally, I would like to point out that the strength of the bargaining channel in determining the wage rate also depends on the worker's bargaining parameter $\xi$. Loosely speaking, the wage rate becomes less sensitive with respect to student loan debt when $\xi$ increases. In the extreme case with $\xi = 1$, the worker's wage rate is always equal to the marginal product of labor $z\rho$, and is therefore not varying with student loan debt at all. When $\xi = 0$, the worker's wage rate is set such that the employment value is equal to the unemployment value. In this case, the sensitivity of wage rate with respect to student loan debt closely depends on the sensitivity of the worker's unemployment value with respect to student loan debt. As a result, the strength of the bargaining channel is comparable to the strength of the liquidity channel of debt burden.

# F Background Information for Federal Student Loan Programs

The U.S. federal student loan programs include the William D. Ford Federal Direct Loan Program, the Federal Family Education Loan (FFEL) Program, and the Federal Perkins Loan Program.

The Direct Loan Program is the largest program whose lender is the U.S. Department of Education. This program includes Direct Subsidized/Unsubsidized Loans (also called Stafford Loans), Direct PLUS Loans, and Direct Consolidation Loans. The FFEL Program was the second largest program, funded through a public/private partnership administered at the state and local level. This program includes Subsidized/Unsubsidized Federal Stafford Loans, Federal PLUS Loans, and Federal Consolidation Loans. Following the passage of the Health Care and Education Reconciliation Act of 2010 on March 26, 2010, the FFEL Program was eliminated, and no subsequent loans were permitted to be made under the program after June 30, 2010.[18] The Perkins Program is a school-based loan program for undergraduate and graduate students with exceptional financial need. Under this program, the school is the lender. The Perkins Program only accounts for about 1% of the outstanding federal student loan debt.

Below, I introduce the main features of federal student loans. More detailed information can be found at Federal Student Aid, Fin Aid, and The Institute For College Access & Success.

## F.1 Grace Period

Under both the Direct Loan Program and the FFEL Program, the borrowers do not have to begin repaying most federal student loans until after they leave college or drop below half-time enrollment. This so-called grace period gives borrowers time to get financially settled and to select their repayment plans. For most loans, interest will accrue during the grace period. However, not all federal student loans have a grace period: Direct Subsidized/Unsubsidized Loans and Subsidized/Unsubsidized Federal Stafford Loans have a 6-month grace period before payments are due. PLUS loans have no grace period. They enter repayment once they are fully disbursed, but may be eligible for a deferment.

## F.2 Consolidation Loan

Federal student loan borrowers can consolidate their federal education loans into a consolidation loan. Loan consolidation can greatly simplify loan repayment by centralizing loans to one bill and can lower monthly payments by giving borrowers up to 30 years to repay their loans. After consolidation, borrowers will be able to switch their variable interest rate loans to a fixed interest rate. The downside of loan

[18] On April 24, 2009, President Barack Obama called for an end to the FFEL program, calling it a wasteful and inefficient system of "taxpayers...paying banks a premium to act as middlemen–a premium that costs the American people billions of dollars each year....a premium we cannot afford."

consolidation is that borrowers might lose benefits offered with the original loans, which may include interest rate discounts, principal rebates, or some loan cancellation benefits.

## F.3 Repayment Plans

Both the Direct Loan Program and the FFEL Program allow borrowers to choose from among different repayment plans: standard repayment plan, graduated repayment plan, extended repayment plan, and income-driven repayment plan. Borrowers can switch among these plans once a year. All federal education loans allow prepayment without penalty. For loans that are not in default, any excess payment is applied first to interest and then to principal.

### F.3.1 Standard Repayment Plan

The default option for federal student loan borrowers is the standard repayment plan. Under this plan, monthly payments are a fixed amount of at least $50 each month and made for up to 10 years for all loan types except Direct Consolidation Loans and FFEL Consolidation Loans. If borrowers have a Direct Consolidation Loan or FFEL Consolidation Loan, the length of the repayment period will depend on the amount of total education loan indebtedness. The maximum repayment period is 30 years.

### F.3.2 Graduated Repayment Plan

Under the graduated repayment plan, monthly payments start out low and increase every two years. The repayment period is 10 years for all loan types except for Direct Consolidation Loans and FFEL Consolidation Loans, which allow an extension of the repayment period to 30 years depending on the amount of total education loan indebtedness. Moreover, monthly payments can at least cover the amount of interest that accrues between payments, and they are never more than three times greater than any other payment.

### F.3.3 Extended Repayment Plan

Under the extended repayment plan, monthly payments are either fixed or graduated. The repayment period can be extended up to 25 years. As a result, monthly payments are generally lower than those made under the standard and graduated repayment plans. To qualify for the extended repayment plan, borrowers must have had no outstanding balance on a Direct Loan/FFEL Loan as of October 7, 1998, or on the date they obtained a Direct Loan/FFEL Loan after October 7, 1998. Moreover, borrowers must have more than $30,000 in outstanding Direct Loans or in FFEL Loans.

### F.3.4 Income-Driven Repayment Plans

The goal of income-driven repayment plans is to help make borrowers' monthly payments more affordable by basing them on their income and family size.

The income-based application now includes four different income-driven repayment plans: income-contingent repayment plan (ICR Plan), income-based repayment plan (IBR Plan), pay as you earn repayment plan (PAYE Plan), and revised pay as you earn repayment plan (REPAYE Plan).

**ICR** ICR is available since 1994 and applies only to the Direct Loan Program. It does not have an eligibility requirement; any borrower with an eligible Direct Loan may choose to repay under the ICR Plan. However, ICR is generally less favorable due to the high repayment ratio. The required monthly payment is the lessor of 20% of the borrower's discretionary income[19] or the amount the borrower would pay under a 12-year fixed repayment plan, multiplied by an "income percentage factor". As a result, the monthly payment could be higher than that under the standard repayment plan. Moreover, ICR only applies to the Direct Loan Program, accounting for less than 30% of outstanding loans during late 90s and early 00s (Lee and Egan, 2009) and 20% during late 00s (The Institute for Higher Education Policy, 2014), which limits its popularity.[20] In fact, between 1996 to 2000, fewer than 1% of borrowers who borrow from Direct Loans choose ICR, although it is 17% of all Direct Loans projected by the Secretary of Education (Schrag, 2001).

**IBR** IBR has been available since July 1, 2009, to borrowers with either Direct or FFEL Loans. To initially enter IBR, borrowers need to have a "partial financial hardship", which means that the payment under IBR is less than the required payment under a standard 10-year repayment plan. Any loan balance that remains after 25 years will be forgiven.[21] The monthly payment under IBR is either 15% of discretionary income or the payment under the standard repayment plan, whichever is smaller. The amendment made by the Health Care and Education Reconciliation Act of 2010 allows borrowers who take out their first loans on or after July 1, 2014, to pay 10% of discretionary income, and the forgiveness period is shortened to 20 years (known as the new IBR plan). The participation rate in loan amounts has been steadily increasing due to better publicity.[22]

[19]For ICR, discretionary income is the difference between adjusted gross income and 100% of the poverty guideline amount, adjusted by family size and state-dependent. This differs from the standard used by other income-driven repayment plans, in which discretionary income is based on 150% of the poverty guideline amount.

[20]Schrag (2001) provides an detailed discussion about the low participation rate in ICR. One important reason is due to the competition between FFEL lenders and the Direct Loan Program. The FFEL lenders eager to preserve billions of dollars of virtually risk-free federally guaranteed profit, bitterly fought against the creation of the federal direct lending program. Because ICR offers a competitive edge to Direct Loans, the FFEL industry targeted that plan for ridicule and attack. In 1996, three industry groups issued a report attacking the plan as too costly for students. It purported to show that income-contingent repayment is "an expensive option", compared to other repayment plans.

[21]The discharged loan amounts are considered as taxable income, a sort of anti-moral-hazard for excessive borrowing.

[22]In fact, the participation rate was low for a few years after the initialization of IBR in 2009. This is because enrolling in the income-based repayment plan requires working with a loan servicer to complete a 12-page application. As shown by the Consumer Financial Protection Bureau, this is often a bumpy process that can take months. In the meantime, the bills keep coming and millions of borrowers end up in default. Government officials had criticized loan servicers for not being aggressive in enrolling borrowers into IBR and for not informing enough borrowers about the initiative. It is possibly due to an incentive misalignment. The reduced monthly payments under IBR could bite into the revenues of companies under the FFEL Program or hurt holders of FFEL Loans that these companies have securitized and sold to investors. For example, an article published on Huffpost Politics criticized Sallie Mae, the nation's largest servicer of federal student loans, for the failure to enroll many of its distressed borrowers into debt relief programs. Another article on U.S. News reported that student debt relief companies profit from borrowers' confusion.

**PAYE** PAYE was introduced on December 21, 2012, and it has similar terms to the new IBR plan. However, it is not available to students with older loans–those who borrowed before September 30, 2007, or those who have not borrowed since September 30, 2011.

**REPAYE** In June 2014, President Obama issued a Presidential Memorandum directing the Department of Education to propose regulations to further ease the burden of student loan debt. On October 27, 2015, the Department of Education issued a final regulation establishing REPAYE. REPAYE improves upon PAYE and extends its protections to all student borrowers with Direct Loans.

Under REPAYE, monthly payments are 10% of discretionary income for all Direct Loan borrowers regardless of their loan origination dates. REPAYE forgives remaining debt after 20 years for those who borrowed only for undergraduate study and 25 years for those who borrowed for graduate study.

**Which Income-Driven Repayment Plan to Choose?** The older ICR is less borrower-friendly, and usually does not deserve a concern. FFEL Loan borrowers who borrowed before September 30, 2007, are only eligible for IBR. Direct Loan borrowers are usually eligible for multiple income-driven repayment plans and face a choice to make.

New Direct Loan borrowers since July 2014 are eligible for IBR, PAYE, and REPAYE. All three offer the same 10% repayment ratio, but PAYE is slightly more favorable than IBR because all outstanding interest is capitalized under IBR but the amount is capped under PAYE. Moreover, borrowers under IBR are required to spend at least one month in the standard repayment plan before switching to a new plan, but borrowers under PAYE face no such hurdle.

Whether PAYE is better than REPAYE depends on individual circumstances and concerns. REPAYE has three main drawbacks: first, borrowers with any Direct Loans obtained for graduate or professional school face a 25-year repayment period under REPAYE, five years longer than the 20-year PAYE period. Second, spousal income is generally included in calculating "discretionary income". Third, REPAYE does not cap monthly payments at the "standard" repayment amount, as under PAYE and IBR. REPAYE has its own advantages: first, all Direct Loan borrowers are eligible. Second, borrowers whose payments are insufficient to cover the interest accrued during the month will only be charged for up to 50% of the unpaid interest, and interest is not capitalized if the borrower no longer has a partial financial hardship. Borrowers who would face negative amortization can thus save on interest under REPAYE.

Borrowers with older Direct Loans may face a choice between REPAYE and the pre-July 2014 IBR formulation. Most will do better under REPAYE as it offers a lower repayment ratio and a shorter repayment period for undergraduate loans. However, REPAYE's inclusion of spousal income and the lack of a payment cap, may nonetheless make IBR a better option.

**Expanding Enrollment in Income-Driven Repayment Plans** The income-driven repayment plans alleviate the burden from inflexible repayment. But due to the reasons above, the enrollment is still quite low. As suggested by The Executive Office of the President of the United States (2016), continuing to expand enrollment in income-driven repayment plans remains a key priority for the administration. In fact, the administration has used several tools to increase enrollment, such as behavioral "nudges", improved loan servicer contract requirements, efforts associated with the President's Student Aid Bill of Rights, a student debt challenge to gather commitments from external stakeholders, and increased and improved targeted outreach to key borrower segments who would benefit from income contingency. The participation rate in income-driven repayment plans has quadrupled over the last four years from 5% in 2012 to 20% in 2016. In April 2016, the administration announced a series of new actions to further expand the enrollment in income-driven repayment plans.

### F.4 Deferment and Forbearance

Under certain circumstances, borrowers can receive a deferment or forbearance that allows them to temporarily postpone or reduce their federal student loan payments. But many borrowers do not use these options because applying for deferment and forbearance involves bureaucratic hurdles and detailed paper work.

A deferment is a period during which repayment of the principal and interest of the loan is temporarily delayed. During a deferment, borrowers do not need to make payments. If loans are Federal Perkins Loan, Direct Subsidized Loan, or Subsidized Federal Stafford Loan, the federal government may pay the interest on the loan during a period of deferment. For other types of loans, borrowers are responsible for paying the interest that accrues during the deferment period; otherwise, it may be capitalized.

Borrowers are eligible for a deferment if they are in college, career schools, graduate fellowship programs, approved rehabilitation training program for the disabled, or active duty military service during a war, military operation, or national emergency. Borrowers are eligible for a deferment for up to 3 years if they are experiencing a period of economic hardship, including a period of unemployment or inability to find full-time employment.

With forbearance, borrowers may be able to stop making payments or reduce their monthly payment for up to 12 months. Interest will continue to accrue on all loan types, including subsidized loans. To be eligible for a mandatory forbearance, borrowers must meet certain criteria including financial hardship, illness, national or teaching services, etc. If none of these criteria are satisfied, borrowers can request a discretionary forbearance, and the lender decides whether to grant forbearance or not.

### F.5 Default and Delinquency

Unlike virtually all other forms of credit, student loans are generally not underwritten: they are frequently offered to young borrowers who have little or no credit history and little to no current income; second, they are accumulated during college studies before individuals enter the job market; and third, lenders (now primarily the taxpayers) are given additional security in that student loans, unlike other forms of debt, are not dischargeable in bankruptcy.[23] This makes the default cost of student loans very high.

[23] Student loans can only be discharged if borrowers prove "undue hardship" through a court determination. The undue hardship standard is generally difficult to meet, making student loans practically non-dischargeable through bankruptcy.

Student loans become delinquent the first day after borrowers miss a payment. The delinquency will continue until all payments are made to bring loans current. After a payment reaches 90 days past due, the delinquent status will be reported to the three major credit bureaus and a negative mark will be added to the borrower's credit report. Default occurs when borrowers are delinquent for 270 days. At this point, the debt will be put into collections and payment will be required from collection agencies.

The consequences of default can be severe, and can include: (1). Loss of eligibility for forgiveness plans. (2). The entire unpaid balance and any interest is immediately due and payable. (3). Borrowers' federal and state taxes may be withheld through a tax offset. (4). Student loan debt will increase because of the late fees, additional interest, court costs, collection fees, attorney's fees, and any other costs associated with the collection process. The total increase in cost could be up to 25% of the unpaid balance. (5). Could result in a wage garnishment of 15% of disposable pay.

Once in default, borrowers can get out of default either by repaying the loan in full or through loan rehabilitation. Loan rehabilitation is a one-time opportunity to clear the default on a defaulted federal education loan and regain eligibility for federal student aid. If borrowers redefault on the loan, they will not be able to rehabilitate the loan a second time. If a judgment has been obtained on the defaulted loan, it is not eligible for rehabilitation. To rehabilitate the Direct Loans or FFEL Loans, borrowers and U.S. Department of Education must agree on a reasonable and affordable payment plan. Generally, a monthly payment is considered to be reasonable and affordable if it is at least 1.0% of the current loan balance.

# G Robustness Check

I conduct three robustness checks for the quantitative results reported in Tables 6-7. In each robustness check, I reestimate all internally-estimated parameters following the procedure in subsection 5.2.3.

## G.1 Risk Aversion

One important parameter that determines the effect of debt burden on job search is risk aversion $\gamma$. In my baseline specification, $\gamma$ is set to be 3 according to the precautionary savings literature. I now reduce its value to 1.5, according to the macro development literature on financial frictions. The simulation results in Table G.5 indicate that with lower risk aversion, the reduction in wage income is 72.3% of the baseline under the fixed repayment plan. IBR alleviates the debt burden by 44.7% and increases wage income by 1.1%, compared to 52.1% and 1.9% in the baseline. 22.7% of the reduction in debt burden is attributed to higher reservation wages as opposed to 37.0% in the baseline.

In general equilibrium (Table G.6), IBR increases college entry rate and overall welfare by 4.2% and 2.0% compared with 6.1% and 2.4% in the baseline. The decomposition shows that increased job postings, college entry rate, and insurance in the labor market contribute to 0.4%, 0.9%, and 0.7% of the welfare increase, compared with 0.5%, 1.1%, and 0.8% in the baseline.

Table G.5: Robustness check (lower risk aversion $\gamma = 1.5$): Evaluation of IBR.

| | **Non-borrowers** | **Normalized borrowers** | | | **Difference** | |
|---|---|---|---|---|---|---|
| | | FIX | IBR | IBR($w^*_{FIX}$) | IBR-FIX | IBR($w^*_{FIX}$)-IBR |
| Compensation ($) | N/A | 5,524 | 2,890 | 3,489 | -2,634 | 599 |
| Unemp. dur. | 24.2 | 22.8 | 23.9 | 22.9 | 1.1 | -1.0 |
| (week) | | (-5.8%) | (-1.2%) | (-5.4%) | (4.6%) | (-4.2%) |
| Match quality | 0.844 | 0.826 | 0.836 | 0.827 | 0.010 | -0.009 |
| | | (-2.1%) | (-0.9%) | (-2.0%) | (1.3%) | (-1.1%) |
| Wage income | 48,234 | 46,782 | 47,340 | 45,912 | 558 | -1,428 |
| ($) | | (-3.0%) | (-1.9%) | (-4.8%) | (1.1%) | (-2.9%) |
| Output | 61,182 | 59,289 | 60,098 | 58,212 | 809 | -1,886 |
| ($) | | (-3.1%) | (-1.8%) | (-4.9%) | (1.3%) | (-3.1%) |
| Labor supply | 1,737 | 1,726 | 1,720 | 1,699 | -6 | -21 |
| (hour) | | (-0.6%) | (-1.0%) | (-2.2%) | (-0.4%) | (-1.2%) |

Table G.6: Robustness check (lower risk aversion $\gamma = 1.5$): General Equilibrium Implications of Student Debt.

| | FIX | IBR | | |
|---|---|---|---|---|
| | | (1) | (2) | (3) |
| Fraction of college graduates | 41.2% | 45.4% | 45.5% | 41.2% |
| Fraction of borrowers | 63.2% | 65.8% | 65.9% | 63.2% |
| Average debt among borrowers ($) | 10,432 | 16,135 | 16,231 | 10,432 |
| Job contact rate | 0.82 | 0.86 | 0.82 | 0.82 |
| Wage income ($) | 37,835 | 38,890 | 38,590 | 38,019 |
| | | (2.8%) | (2.0%) | (0.5%) |
| Output ($) | 46,593 | 47,396 | 47,107 | 46,811 |
| | | (1.7%) | (1.1%) | (0.5%) |
| Welfare (%) | | 2.0% | 1.6% | 0.7% |

## G.2 Elasticity of Labor Supply

The elasticity of labor supply determines the distortion of IBR on the number of hours. In my baseline specification, $\sigma$ is set to be 2.59 so that the tax-modified Frisch elasticity is 0.33. I now check the model's implication by setting $\sigma = 0.78$ and $\sigma = 88.89$, corresponding to 1 and 0.01 tax-modified labor supply elasticities.

When elasticity is 1, the simulation results in Table G.7 indicate that IBR barely alleviates the debt burden or increases wage income due to the large distortion on labor supply. Borrowers' labor supply is on average reduced by 3.5% relative to non-borrowers, compared to 1.5% in the baseline. The reservation wage effect is still positive, as the wealth compensation would increase by $1,100 for borrowers if reservation wages are fixed. As shown in Table G.8, there is not much response in general equilibrium as the debt burden is almost unchanged.

When elasticity is 0.01, there is almost no response in labor supply when borrowers switch to IBR. As a result, IBR becomes very effective in alleviating the debt burden. The wealth compensation is reduced by 56.4% on average when all borrowers switch to IBR, compared to 52.1% in the baseline. In general equilibrium (G.10), IBR increases college entry rate and overall welfare by 6.6% and 2.6% compared with 6.1% and 2.4% in the baseline. The decomposition shows that increased job postings, college entry rate, and insurance in the labor market contribute to 0.6%, 1.1%, and 0.9% of the welfare increase, compared with 0.5%, 1.1%, and 0.8% in the baseline.

Table G.7: Robustness check (higher labor supply elasticity $\sigma = 0.78$): Evaluation of IBR.

| | **Non-borrowers** | **Normalized borrowers** | | | **Difference** | |
|---|---|---|---|---|---|---|
| | | FIX | IBR | IBR($w^*_{FIX}$) | IBR-FIX | IBR($w^*_{FIX}$)-IBR |
| Compensation ($) | N/A | 6,530 | 6,412 | 7,512 | -118 | 1,100 |
| Unemp. dur. | 24.0 | 22.1 | 23.5 | 22.4 | 1.4 | -1.1 |
| (week) | | (-7.9%) | (-2.1%) | (-6.7%) | (5.8%) | (-4.6%) |
| Match quality | 0.877 | 0.858 | 0.869 | 0.857 | 0.011 | -0.012 |
| | | (-2.2%) | (-0.9%) | (-2.3%) | (1.3%) | (-1.4%) |
| Wage income | 48,121 | 45,976 | 46,012 | 44,545 | 36 | -1,467 |
| ($) | | (-4.5%) | (-4.4%) | (-7.4%) | (0.1%) | (-3.0%) |
| Output | 56,387 | 53,887 | 53,940 | 52,173 | 53 | -1,677 |
| ($) | | (-4.4%) | (-4.3%) | (-7.5%) | (0.1%) | (-3.0%) |
| Labor supply | 1,657 | 1,621 | 1,599 | 1,575 | -22 | -24 |
| (hour) | | (-2.2%) | (-3.5%) | (-4.9%) | (-1.3%) | (-1.4%) |

Table G.8: Robustness check (higher labor supply elasticity $\sigma = 0.78$): General Equilibrium Implications of Student Debt.

| | **FIX** | **IBR** | | |
|---|---|---|---|---|
| | | (1) | (2) | (3) |
| Fraction of college graduates | 41.3% | 41.6% | 41.6% | 41.7% |
| Fraction of borrowers | 62.0% | 62.2% | 62.2% | 62.3% |
| Average debt among borrowers ($) | 10,315 | 10,539 | 10,578 | 10,315 |
| Job contact rate | 0.82 | 0.82 | 0.82 | 0.82 |
| Wage income ($) | 37,489 | 37,635 | 37,603 | 37,521 |
| | | (0.4%) | (0.3%) | (0.1%) |
| Output ($) | 42,529 | 42,647 | 42,613 | 42,589 |
| | | (0.3%) | (0.2%) | (0.1%) |
| Welfare (%) | | 0.3% | 0.2% | 0.1% |

Table G.9: Robustness check (lower labor supply elasticity $\sigma = 88.89$): Evaluation of IBR.

| | **Non-borrowers** | **Normalized borrowers** | | | **Difference** | |
|---|---|---|---|---|---|---|
| | | FIX | IBR | IBR($w^*_{FIX}$) | IBR-FIX | IBR($w^*_{FIX}$)-IBR |
| Compensation ($) | N/A | 6,512 | 2,842 | 4,237 | -3,670 | 1,395 |
| Unemp. dur. | 23.7 | 22.0 | 23.5 | 22.2 | 1.5 | -1.3 |
| (week) | | (-7.2%) | (-0.8%) | (-6.3%) | (6.4%) | (-5.5%) |
| Match quality | 0.811 | 0.787 | 0.807 | 0.788 | 0.020 | -0.019 |
| | | (-3.0%) | (-0.5%) | (-2.8%) | (2.5%) | (-2.3%) |
| Wage income | 46,547 | 45,198 | 46,210 | 45,179 | 1,021 | -1,031 |
| ($) | | (-2.9%) | (-0.7%) | (-2.9%) | (2.2%) | (-2.2%) |
| Output | 64,628 | 62,685 | 64,256 | 62,730 | 1,571 | -1,526 |
| ($) | | (-3.0%) | (-0.6%) | (-2.9%) | (2.4%) | (-2.3%) |
| Labor supply | 1,732 | 1,732 | 1,731 | 1,731 | -1 | 0 |
| (hour) | | (-0.0%) | (-0.1%) | (-0.1%) | (-0.1%) | (-0.0%) |

Table G.10: Robustness check (lower labor supply elasticity $\sigma = 88.89$): General Equilibrium Implications of Student Debt.

| | **FIX** | **IBR** | | |
|---|---|---|---|---|
| | | (1) | (2) | (3) |
| Fraction of college graduates | 41.4% | 48.0% | 48.2% | 41.4% |
| Fraction of borrowers | 62.2% | 67.8% | 67.9% | 62.2% |
| Average debt among borrowers ($) | 10,451 | 17,865 | 18,010 | 10,451 |
| Job contact rate | 0.82 | 0.89 | 0.82 | 0.82 |
| Wage income ($) | 36,423 | 37,765 | 37,342 | 36,679 |
| | | (3.7%) | (2.5%) | (0.7%) |
| Output ($) | 48,138 | 49.264 | 48,996 | 48,425 |
| | | (2.3%) | (1.8%) | (0.6%) |
| Welfare (%) | | 2.6% | 2.0% | 0.9% |

### G.3 Access to Other Credit

Credit access alleviates the liquidity problem, which would attenuate the effect of debt burden on job search. In the baseline specification, agents cannot borrow. I now relax this assumption. Using data from the Survey of Consumer Finances (SCF), Kaplan and Violante (2014) estimate that the median ratio of credit limit to annual labor income is 18.5% for households aged 22 to 59. Based on this estimate, I allow employed agents to borrow 18.5% of their wage income, and unemployed agents to borrow 18.5% of UI benefits (i.e., $1,500). The simulation results in Table G.11 indicate that credit access slightly alleviates the debt burden. The reduction in wage income is 90% of the baseline under the fixed repayment plan. The small difference comes from the fact agents cannot borrow much due to the low income during unemployment. IBR alleviates the debt burden by 45.8% and increases wage income by 1.8%, compared to 52.1% and 1.9% in the baseline. 27.6% of the reduction in debt burden is attributed to higher reservation wages as opposed to 37.0% in the baseline.

In general equilibrium (Table G.12), IBR increases college entry rate and overall welfare by 5.4% and 2.2% compared with 6.1% and 2.4% in the baseline. The decomposition shows that increased job postings, college entry rate, and insurance in the labor market contribute to 0.5%, 1.0%, and 0.7% of the welfare increase, compared with 0.5%, 1.1%, and 0.8% in the baseline.

Table G.11: Robustness check (credit access): Evaluation of IBR.

| | **Non-borrowers** | **Normalized borrowers** | | | **Difference** | |
|---|---|---|---|---|---|---|
| | | FIX | IBR | IBR($w^*_{FIX}$) | IBR-FIX | IBR($w^*_{FIX}$)-IBR |
| Compensation ($) | N/A | 7,623 | 4,135 | 5,098 | -3,488 | 963 |
| Unemp. dur. | 24.0 | 22.5 | 23.6 | 22.5 | 1.1 | -1.1 |
| (week) | | (-6.3%) | (-1.7%) | (-6.3%) | (4.6%) | (-4.6%) |
| Match quality | 0.840 | 0.817 | 0.831 | 0.818 | 0.014 | -0.013 |
| | | (-2.7%) | (-1.1%) | (-2.6%) | (1.6%) | (-1.5%) |
| Wage income | 47,356 | 45,550 | 46,399 | 44,976 | 849 | -1,423 |
| ($) | | (-3.8%) | (-2.0%) | (-5.0%) | (1.8%) | (-3.0%) |
| Output | 59,489 | 57,343 | 58,212 | 56,562 | 869 | -1,650 |
| ($) | | (-3.6%) | (-2.1%) | (-4.9%) | (1.5%) | (-2.8%) |
| Labor supply | 1,722 | 1,712 | 1,705 | 1,695 | -7 | -10 |
| (hour) | | (-0.6%) | (-1.0%) | (-1.6%) | (-0.4%) | (-0.6%) |

Table G.12: Robustness check (credit access): General Equilibrium Implications of Student Debt.

| | **FIX** | **IBR** | | |
|---|---|---|---|---|
| | | (1) | (2) | (3) |
| Fraction of college graduates | 41.3% | 46.7% | 46.7% | 41.3% |
| Fraction of borrowers | 62.1% | 66.9% | 67.0% | 62.1% |
| Average debt among borrowers ($) | 10,333 | 16,894 | 16,978 | 10,333 |
| Job contact rate | 0.82 | 0.87 | 0.82 | 0.82 |
| Wage income ($) | 37,015 | 38,124 | 37,764 | 37,203 |
| | | (3.0%) | (2.0%) | (0.5%) |
| Output ($) | 45,198 | 46,026 | 45,845 | 45,367 |
| | | (1.8%) | (1.4%) | (0.4%) |
| Welfare (%) | | 2.2% | 1.7% | 0.7% |

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